

# ANALYSIS OF THE $\frac{1}{2}^{\pm}$ PENTAQUARK STATES IN THE DIQUARK-DIQUARK-ANTIQUARK MODEL WITH QCD SUM RULES

Zhi-Gang Wang<sup>1</sup>

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

## Abstract

In this article, we construct both the axialvector-diquark-axialvector-diquark-antiquark type and axialvector-diquark-scalar-diquark-antiquark type interpolating currents, then calculate the contributions of the vacuum condensates up to dimension-10 in the operator product expansion, and study the masses and pole residues of the  $J^P = \frac{1}{2}^{\pm}$  hidden-charm pentaquark states with the QCD sum rules in a systematic way. In calculations, we use the formula  $\mu = \sqrt{M_P^2 - (2M_c)^2}$  to determine the energy scales of the QCD spectral densities. We take into account the  $SU(3)$  breaking effects of the light quarks, and obtain the masses of the hidden-charm pentaquark states with the strangeness  $S = 0, -1, -2, -3$ , which can be confronted with the experimental data in the future.

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Key words: Pentaquark states, QCD sum rules

## 1 Introduction

Recently, the LHCb collaboration studied the  $\Lambda_b^0 \rightarrow J/\psi K^- p$  decays, and observed two pentaquark candidates  $P_c(4380)$  and  $P_c(4450)$  in the  $J/\psi p$  mass spectrum with the significances of more than  $9\sigma$  [1]. The measured masses and widths are  $M_{P_c(4380)} = 4380 \pm 8 \pm 29$  MeV,  $M_{P_c(4450)} = 4449.8 \pm 1.7 \pm 2.5$  MeV,  $\Gamma_{P_c(4380)} = 205 \pm 18 \pm 86$  MeV, and  $\Gamma_{P_c(4450)} = 39 \pm 5 \pm 19$  MeV, respectively. The decays  $P_c(4380) \rightarrow J/\psi p$  take place through relative S-wave while the decays  $P_c(4450) \rightarrow J/\psi p$  take place through relative P-wave, the decays  $P_c(4380) \rightarrow J/\psi p$  are kinematically favored and the  $P_c(4380)$  has larger width. The preferred spin-parity of the  $P_c(4380)$  and  $P_c(4450)$  are  $J^P = \frac{3}{2}^-$  and  $\frac{5}{2}^+$ , respectively. There have been several attempted assignments, such as the  $\Sigma_c \bar{D}^*$ ,  $\Sigma_c^* \bar{D}^*$ ,  $\chi_{c1} p$ ,  $J/\psi N(1440)$ ,  $J/\psi N(1520)$  molecule-like pentaquark states [2] (or not the molecular pentaquark states [3]), the diquark-diquark-antiquark type pentaquark states [4, 5, 6, 7, 8, 9, 10], the diquark-triquark type pentaquark states [11], re-scattering effects [12], etc.

The quarks have color  $SU(3)$  symmetry, we can construct the pentaquark configurations according to the routine quark  $\rightarrow$  diquark  $\rightarrow$  pentaquark,

$$(3 \otimes 3) \otimes (3 \otimes 3) \otimes \bar{3} = (\bar{3} \oplus 6) \otimes (\bar{3} \oplus 6) \otimes \bar{3} = \bar{3} \otimes \bar{3} \otimes \bar{3} \oplus \dots = 1 \oplus \dots, \quad (1)$$

where the 1, 3 ( $\bar{3}$ ) and 6 denote the color singlet, triplet (antitriplet) and sextet, respectively. The diquarks  $q_j^T C \Gamma q_k'$  have five structures in Dirac spinor space, where  $C \Gamma = C \gamma_5$ ,  $C$ ,  $C \gamma_\mu \gamma_5$ ,  $C \gamma_\mu$  and  $C \sigma_{\mu\nu}$  for the scalar, pseudoscalar, vector, axialvector and tensor diquarks, respectively, and the  $j$  and  $k$  are color indexes. The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet  $\bar{3}_c$ , flavor antitriplet  $\bar{3}_f$  and spin singlet  $1_s$  [13], while the favored configurations are the scalar ( $C \gamma_5$ ) and axialvector ( $C \gamma_\mu$ ) diquark states [14, 15]. The calculations based on the QCD sum rules indicate that the heavy-light scalar and axialvector diquark states have almost degenerate masses [14], while the masses of the light axialvector diquark states lie (150 – 200) MeV above that of the light scalar diquark states [15], if they have the same quark constituents.

In Refs.[7, 8], we choose the light scalar diquark and heavy axialvector diquark (or heavy scalar diquark) as the basic constituents, construct both the scalar-diquark-axialvector-diquark-antiquark type and scalar-diquark-scalar-diquark-antiquark type interpolating currents, which are supposed to couple potentially to the lowest pentaquark states according to the light scalar diquark

<sup>1</sup>E-mail: zgwang@aliyun.com.

constituent [15], then calculate the contributions of the vacuum condensates up to dimension-10 in the operator product expansion and study the masses and pole residues of the  $J^P = \frac{3}{2}^-$ ,  $\frac{5}{2}^+$  and  $\frac{1}{2}^\pm$  hidden-charm pentaquark states with the QCD sum rules. The numerical results favor assigning the  $P_c(4380)$  and  $P_c(4450)$  to be the  $\frac{3}{2}^-$  and  $\frac{5}{2}^+$  diquark-diquark-antiquark type pentaquark states, respectively [7]. In Ref.[7], we take the energy scale formula  $\mu = \sqrt{M_P^2 - (2M_c)^2}$  to determine the energy scales of the QCD spectral densities, the resulting pole contributions are about (40 – 60)%, and the contributions of the vacuum condensates of dimension 10 are less than 5%, the two criteria (pole dominance at the phenomenological side and convergence of the operator product expansion) of the conventional QCD sum rules can be satisfied. Now we extend our previous work to study the  $\frac{1}{2}^\pm$  hidden-charm pentaquark states in a systematic way, where the energy scale formula  $\mu = \sqrt{M_P^2 - (2M_c)^2}$  serves as an additional constraint on the predicted masses. All the predictions can be confronted with the experimental data in the future, and the assignments of the  $P_c(4380)$  and  $P_c(4450)$  in the scenario of the diquark-diquark-antiquark type pentaquark states can be testified.

In this article, we take the light axialvector diquarks and heavy axialvector diquarks (and heavy scalar diquarks) as the basic constituents, and study the axialvector-diquark-axialvector-diquark-antiquark type and axialvector-diquark-scalar-diquark-antiquark type pentaquark configurations. Now we illustrate how to construct the pentaquark configurations in the diquark-diquark-antiquark model according to the spin-parity  $J^P$ ,

$$1_{q_1 q_2}^+ \otimes 0_{q_3 c}^+ \otimes \frac{1}{2}_{\bar{c}}^- = \frac{1}{2}_{\underline{q_1 q_2 q_3 c \bar{c}}}^- \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^-, \quad (2)$$

$$\begin{aligned} 1_{q_1 q_2}^+ \otimes 1_{q_3 c}^+ \otimes \frac{1}{2}_{\bar{c}}^- &= [0_{q_1 q_2 q_3 c}^+ \oplus 1_{q_1 q_2 q_3 c}^+ \oplus 2_{q_1 q_2 q_3 c}^+] \otimes \frac{1}{2}_{\bar{c}}^- \\ &= \frac{1}{2}_{\underline{q_1 q_2 q_3 c \bar{c}}}^- \oplus \left[ \frac{1}{2}_{q_1 q_2 q_3 c \bar{c}}^- \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^- \right] \oplus \left[ \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^- \oplus \frac{5}{2}_{q_1 q_2 q_3 c \bar{c}}^- \right], \end{aligned} \quad (3)$$

$$\begin{aligned} 1_{q_1 q_2}^+ \otimes 0_{q_3 c}^+ \otimes \left[ 1^- \otimes \frac{1}{2}_{\bar{c}}^- \right] &= 1_{q_1 q_2}^+ \otimes 0_{q_3 c}^+ \otimes \left[ \frac{1}{2}_{\bar{c}}^+ \oplus \frac{3}{2}_{\bar{c}}^+ \right] \\ &= \left[ \frac{1}{2}_{\underline{q_1 q_2 q_3 c \bar{c}}}^+ \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \right] \oplus \left[ \frac{1}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{5}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \right], \end{aligned} \quad (4)$$

$$\begin{aligned} 1_{q_1 q_2}^+ \otimes 1_{q_3 c}^+ \otimes \left[ 1^- \otimes \frac{1}{2}_{\bar{c}}^- \right] &= [0_{q_1 q_2 q_3 c}^+ \oplus 1_{q_1 q_2 q_3 c}^+ \oplus 2_{q_1 q_2 q_3 c}^+] \otimes \left[ \frac{1}{2}_{\bar{c}}^+ \oplus \frac{3}{2}_{\bar{c}}^+ \right] \\ &= \frac{1}{2}_{\underline{q_1 q_2 q_3 c \bar{c}}}^+ \oplus \left[ \frac{1}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \right] \oplus \left[ \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{5}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \right] \\ &\quad \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \left[ \frac{1}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{5}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \right] \\ &\quad \oplus \left[ \frac{1}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{3}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{5}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \oplus \frac{7}{2}_{q_1 q_2 q_3 c \bar{c}}^+ \right], \end{aligned} \quad (5)$$

where the  $1^-$  denotes the contribution of the additional P-wave to the spin-parity, the subscripts  $q_1 q_2, q_3 c, \dots$  denote the quark constituents. The quark and antiquark have opposite parity, we usually take it for granted that the quarks have positive parity while the antiquarks have negative parity, so the  $\bar{c}$ -quark has  $J^P = \frac{1}{2}^-$ .

In this article, we study the pentaquark states with the underlined spin-parity  $\frac{1}{2}_{\underline{q_1 q_2 q_3 c \bar{c}}}^\pm$ , which are supposed to be the lowest pentaquark states with the light axialvector diquarks. We construct both the axialvector-diquark-axialvector-diquark-antiquark type and axialvector-diquark-scalar-diquark-antiquark type currents  $J(x) = J_{q_1 q_2 q_3}^{j_L j_H}(x)$  according to Eqs.(2-3), where the superscripts  $j_L$  and  $j_H$  denote the spins of the light diquark and heavy diquark, respectively, the subscripts  $q_1,$

$q_2, q_3$  denote the light quark constituents. We calculate the vacuum condensate up to dimension-10 in the operator product expansion, and study the masses and pole residues of the lowest pentaquark states in a systematic way.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the  $\frac{1}{2}^\pm$  pentaquark states in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

## 2 QCD sum rules for the $\frac{1}{2}^\pm$ pentaquark states

We write down the two-point correlation functions  $\Pi_{q_1 q_2 q_3}^{jLjH}(p)$  in the QCD sum rules,

$$\Pi_{q_1 q_2 q_3}^{jLjH}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_{q_1 q_2 q_3}^{jLjH}(x) \bar{J}_{q_1 q_2 q_3}^{jLjH}(0) \} | 0 \rangle, \quad (6)$$

where

$$\begin{aligned} J_{uuu}^{11}(x) &= \varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn} u_j^T(x) C \gamma_\mu u_k(x) u_m^T(x) C \gamma^\mu c_n(x) C \bar{c}_a^T(x), \\ J_{uud}^{11}(x) &= \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} [u_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma^\mu c_n(x) + 2u_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma^\mu c_n(x)] C \bar{c}_a^T(x), \\ J_{udd}^{11}(x) &= \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} [d_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma^\mu c_n(x) + 2d_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma^\mu c_n(x)] C \bar{c}_a^T(x), \\ J_{ddd}^{11}(x) &= \varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn} d_j^T(x) C \gamma_\mu d_k(x) d_m^T(x) C \gamma^\mu c_n(x) C \bar{c}_a^T(x), \end{aligned} \quad (7)$$

$$\begin{aligned} J_{uus}^{11}(x) &= \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} [u_j^T(x) C \gamma_\mu u_k(x) s_m^T(x) C \gamma^\mu c_n(x) + 2u_j^T(x) C \gamma_\mu s_k(x) u_m^T(x) C \gamma^\mu c_n(x)] C \bar{c}_a^T(x), \\ J_{uds}^{11}(x) &= \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} [u_j^T(x) C \gamma_\mu d_k(x) s_m^T(x) C \gamma^\mu c_n(x) + u_j^T(x) C \gamma_\mu s_k(x) d_m^T(x) C \gamma^\mu c_n(x) \\ &\quad + d_j^T(x) C \gamma_\mu s_k(x) u_m^T(x) C \gamma^\mu c_n(x)] C \bar{c}_a^T(x), \\ J_{dds}^{11}(x) &= \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} [d_j^T(x) C \gamma_\mu d_k(x) s_m^T(x) C \gamma^\mu c_n(x) + 2d_j^T(x) C \gamma_\mu s_k(x) d_m^T(x) C \gamma^\mu c_n(x)] C \bar{c}_a^T(x), \end{aligned} \quad (8)$$

$$\begin{aligned} J_{uss}^{11}(x) &= \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} [s_j^T(x) C \gamma_\mu s_k(x) u_m^T(x) C \gamma^\mu c_n(x) + 2s_j^T(x) C \gamma_\mu u_k(x) s_m^T(x) C \gamma^\mu c_n(x)] C \bar{c}_a^T(x), \\ J_{dss}^{11}(x) &= \frac{\varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn}}{\sqrt{3}} [s_j^T(x) C \gamma_\mu s_k(x) d_m^T(x) C \gamma^\mu c_n(x) + 2s_j^T(x) C \gamma_\mu d_k(x) s_m^T(x) C \gamma^\mu c_n(x)] C \bar{c}_a^T(x), \end{aligned} \quad (9)$$

$$J_{sss}^{11}(x) = \varepsilon^{ila} \varepsilon^{ijk} \varepsilon^{lmn} s_j^T(x) C \gamma_\mu s_k(x) s_m^T(x) C \gamma^\mu c_n(x) C \bar{c}_a^T(x), \quad (10)$$

$$\begin{aligned}
J_{uuu}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} u_j^T(x) C \gamma_\mu u_k(x) u_m^T(x) C \gamma_5 c_n(x) \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \\
J_{uud}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} [u_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma_5 c_n(x) + 2u_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma_5 c_n(x)] \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \\
J_{udd}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} [d_j^T(x) C \gamma_\mu d_k(x) u_m^T(x) C \gamma_5 c_n(x) + 2d_j^T(x) C \gamma_\mu u_k(x) d_m^T(x) C \gamma_5 c_n(x)] \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \\
J_{ddd}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} d_j^T(x) C \gamma_\mu d_k(x) d_m^T(x) C \gamma_5 c_n(x) \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \tag{11}
\end{aligned}$$

$$\begin{aligned}
J_{uus}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} [u_j^T(x) C \gamma_\mu u_k(x) s_m^T(x) C \gamma_5 c_n(x) + 2u_j^T(x) C \gamma_\mu s_k(x) u_m^T(x) C \gamma_5 c_n(x)] \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \\
J_{uds}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} [u_j^T(x) C \gamma_\mu d_k(x) s_m^T(x) C \gamma_5 c_n(x) + u_j^T(x) C \gamma_\mu s_k(x) d_m^T(x) C \gamma_5 c_n(x) \\
&\quad + d_j^T(x) C \gamma_\mu s_k(x) u_m^T(x) C \gamma_5 c_n(x)] \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \\
J_{dss}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} [d_j^T(x) C \gamma_\mu d_k(x) s_m^T(x) C \gamma_5 c_n(x) + 2d_j^T(x) C \gamma_\mu s_k(x) d_m^T(x) C \gamma_5 c_n(x)] \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \tag{12}
\end{aligned}$$

$$\begin{aligned}
J_{uss}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} [s_j^T(x) C \gamma_\mu s_k(x) u_m^T(x) C \gamma_5 c_n(x) + 2s_j^T(x) C \gamma_\mu u_k(x) s_m^T(x) C \gamma_5 c_n(x)] \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \\
J_{dss}^{10}(x) &= \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} [s_j^T(x) C \gamma_\mu s_k(x) d_m^T(x) C \gamma_5 c_n(x) + 2s_j^T(x) C \gamma_\mu d_k(x) s_m^T(x) C \gamma_5 c_n(x)] \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \tag{13}
\end{aligned}$$

$$J_{sss}^{10}(x) = \frac{\varepsilon^{ila}\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{3}} s_j^T(x) C \gamma_\mu s_k(x) s_m^T(x) C \gamma_5 c_n(x) \gamma_5 \gamma^\mu C \bar{c}_a^T(x), \tag{14}$$

the  $i, j, k, l, m, n$  and  $a$  are color indices, the  $C$  is the charge conjugation matrix. In the currents  $J_{q_1 q_2 q_3}^{10}(x)$ , the light axialvector diquark combines with the heavy scalar diquark to form a tetraquark with  $J^P = 1^+$  in color triplet according to Eq.(2), while in the currents  $J_{q_1 q_2 q_3}^{11}(x)$ , the light axialvector diquark combines with the heavy axialvector diquark to form a tetraquark with  $J^P = 0^+$  in color triplet according to Eq.(3). Then they couple with the antiquark to form pentaquark states with  $J^P = \frac{1}{2}^-$  in color singlet. In this article, we take the isospin limit, and classify the currents couple to the pentaquark states with degenerate masses into the following 8 types,

$$\begin{aligned}
&J_{uuu,\mu}^{11}(x), J_{uud,\mu}^{11}(x), J_{udd,\mu}^{11}(x), J_{ddd,\mu}^{11}(x); \\
&J_{uus,\mu}^{11}(x), J_{uds,\mu}^{11}(x), J_{dds,\mu}^{11}(x); \\
&J_{uss,\mu}^{11}(x), J_{dss,\mu}^{11}(x); \\
&J_{sss,\mu}^{11}(x); \\
&J_{uuu,\mu}^{10}(x), J_{uud,\mu}^{10}(x), J_{udd,\mu}^{10}(x), J_{ddd,\mu}^{10}(x); \\
&J_{uus,\mu}^{10}(x), J_{uds,\mu}^{10}(x), J_{dds,\mu}^{10}(x); \\
&J_{uss,\mu}^{10}(x), J_{dss,\mu}^{10}(x); \\
&J_{sss,\mu}^{10}(x). \tag{15}
\end{aligned}$$

In calculations, we choose the first current in each type.

The currents  $J_{q_1 q_2 q_3}^{j_L j_H}(0)$  have negative parity, and couple potentially to the  $\frac{1}{2}^-$  hidden-charm pentaquark states  $P_{q_1 q_2 q_3}^{j_L j_H \frac{1}{2}}(\frac{1}{2}^-)$ , where the superscript  $\frac{1}{2}$  denotes the total angular momentum of the anti- $c$ -quark  $\bar{c}$ ,

$$\langle 0 | J_{q_1 q_2 q_3}^{j_L j_H}(0) | P_{q_1 q_2 q_3}^{j_L j_H \frac{1}{2}}(\frac{1}{2}^-)(p) \rangle = \lambda_P^- U^-(p, s), \quad (16)$$

the  $\lambda_P^-$  are the pole residues, the spinors  $U^-(p, s)$  satisfy the Dirac equations  $(\not{p} - M_{P,-})U^-(p, s) = 0$ , the  $s$  are the polarization vectors of the pentaquark states. On the other hand, the currents  $J_{q_1 q_2 q_3}^{j_L j_H}(0)$  also couple potentially to the  $\frac{1}{2}^+$  hidden-charm pentaquark states  $P_{q_1 q_2 q_3}^{j_L j_H \frac{1}{2}}(\frac{1}{2}^+)$  as multiplying  $i\gamma_5$  to the currents  $J_{q_1 q_2 q_3}^{j_L j_H}(x)$  change their parity,

$$\langle 0 | J_{q_1 q_2 q_3}^{j_L j_H}(0) | P_{q_1 q_2 q_3}^{j_L j_H \frac{1}{2}}(\frac{1}{2}^+)(p) \rangle = \lambda_P^+ i\gamma_5 U^+(p, s), \quad (17)$$

the spinors  $U^\pm(p, s)$  (pole residues  $\lambda_i^\pm$ ) have analogous properties [16, 17, 18, 19]. We can study the  $J^P = \frac{1}{2}^+$  hidden-charm pentaquark states without introducing the additional P-wave explicitly, see Eqs.(4-5).

We insert a complete set of intermediate pentaquark states with the same quantum numbers as the current operators  $J(x)$  and  $i\gamma_5 J(x)$  into the correlation functions  $\Pi_{q_1 q_2 q_3}^{j_L j_H}(p)$  to obtain the hadronic representation [20, 21]. After isolating the pole terms of the lowest states of the hidden-charm pentaquark states, we obtain the following results:

$$\Pi_{q_1 q_2 q_3}^{j_L j_H}(p) = \lambda_P^{-2} \frac{\not{p} + M_{P,-}}{M_{P,-}^2 - p^2} + \lambda_P^{+2} \frac{\not{p} - M_{P,+}}{M_{P,+}^2 - p^2} + \dots, \quad (18)$$

where the  $M_\pm$  are the masses of the lowest pentaquark states with the parity  $\pm$ , respectively.

Now we obtain the hadronic spectral densities through the dispersion relation,

$$\begin{aligned} \frac{\text{Im}\Pi_{q_1 q_2 q_3}^{j_L j_H}(s)}{\pi} &= \not{p} \left[ \lambda_P^{-2} \delta(s - M_{P,-}^2) + \lambda_P^{+2} \delta(s - M_{P,+}^2) \right] \\ &\quad + \left[ M_{P,-} \lambda_P^{-2} \delta(s - M_{P,-}^2) - M_{P,+} \lambda_P^{+2} \delta(s - M_{P,+}^2) \right], \\ &= \not{p} \rho_{q_1 q_2 q_3, H}^{j_L j_H, 1}(s) + \rho_{q_1 q_2 q_3, H}^{j_L j_H, 0}(s), \end{aligned} \quad (19)$$

where the subscript index  $H$  denotes the hadron side, then we introduce the weight function  $\exp(-\frac{s}{T^2})$  to obtain the QCD sum rules at the hadron side,

$$\int_{4m_c^2}^{s_0} ds \left[ \sqrt{s} \rho_{q_1 q_2 q_3, H}^{j_L j_H, 1}(s) + \rho_{q_1 q_2 q_3, H}^{j_L j_H, 0}(s) \right] \exp\left(-\frac{s}{T^2}\right) = 2M_{P,-} \lambda_P^{-2} \exp\left(-\frac{M_{P,-}^2}{T^2}\right), \quad (20)$$

$$\int_{4m_c^2}^{s_0} ds \left[ \sqrt{s} \rho_{q_1 q_2 q_3, H}^{j_L j_H, 1}(s) - \rho_{q_1 q_2 q_3, H}^{j_L j_H, 0}(s) \right] \exp\left(-\frac{s}{T^2}\right) = 2M_{P,+} \lambda_P^{+2} \exp\left(-\frac{M_{P,+}^2}{T^2}\right), \quad (21)$$

where the  $s_0$  are the continuum threshold parameters and the  $T^2$  are the Borel parameters. We separate the contributions of the negative-parity (positive-parity) pentaquark states from the positive-parity (negative-parity) pentaquark states explicitly.

In the following, we briefly outline the operator product expansion for the correlation functions  $\Pi_{q_1 q_2 q_3}^{j_L j_H}(p)$  in perturbative QCD. Firstly, we contract the  $u$ ,  $s$  and  $c$  quark fields in the correlation functions  $\Pi_{q_1 q_2 q_3}^{j_L j_H}(p)$  with Wick theorem, and obtain the results:

$$\begin{aligned} \Pi_{uuu}^{11}(p) &= -i \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} C C_{a'a}^T(-x) C \\ &\quad \{ 2Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{jj'}^T(x) C] Tr [\gamma^\mu C_{nn'}(x) \gamma^\nu C U_{mm'}^T(x) C] \\ &\quad - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C U_{jm'}^T(x) C] \}, \end{aligned} \quad (22)$$

$$\begin{aligned}
\Pi_{uus}^{11}(p) = & -\frac{i}{3} \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} C C_{a'a}^T(-x) C \\
& \{ 2Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{jj'}^T(x) C] Tr [\gamma^\mu C_{nn'}(x) \gamma^\nu C S_{mm'}^T(x) C] \\
& + 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C U_{jj'}^T(x) C] Tr [\gamma^\mu C_{nn'}(x) \gamma^\nu C U_{mm'}^T(x) C] \\
& - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C S_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C U_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C U_{jm'}^T(x) C] \} , \tag{23}
\end{aligned}$$

$$\begin{aligned}
\Pi_{uss}^{11}(p) = & -\frac{i}{3} \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} C C_{a'a}^T(-x) C \\
& \{ 2Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{jj'}^T(x) C] Tr [\gamma^\mu C_{nn'}(x) \gamma^\nu C U_{mm'}^T(x) C] \\
& + 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C S_{jj'}^T(x) C] Tr [\gamma^\mu C_{nn'}(x) \gamma^\nu C S_{mm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C U_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C S_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C S_{jm'}^T(x) C] \} , \tag{24}
\end{aligned}$$

$$\begin{aligned}
\Pi_{sss}^{11}(p) = & -i \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} C C_{a'a}^T(-x) C \\
& \{ 2Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{jj'}^T(x) C] Tr [\gamma^\mu C_{nn'}(x) \gamma^\nu C S_{mm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma^\mu C_{nn'}(x) \gamma^\nu C S_{jm'}^T(x) C] \} , \tag{25}
\end{aligned}$$

$$\begin{aligned}
\Pi_{uuu}^{10}(p) = & -i \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} \gamma_5 \gamma^\mu C C_{a'a}^T(-x) C \gamma^\nu \gamma_5 \\
& \{ 2Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{jj'}^T(x) C] Tr [\gamma_5 C_{nn'}(x) \gamma_5 C U_{mm'}^T(x) C] \gamma_5 \gamma^\mu \\
& - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C U_{jm'}^T(x) C] \} , \tag{26}
\end{aligned}$$

$$\begin{aligned}
\Pi_{uus}^{10}(p) = & -\frac{i}{3} \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} \gamma_5 \gamma^\mu C C_{a'a}^T(-x) C \gamma^\nu \gamma_5 \\
& \{ 2Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{jj'}^T(x) C] Tr [\gamma_5 C_{nn'}(x) \gamma_5 C S_{mm'}^T(x) C] \\
& + 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C U_{jj'}^T(x) C] Tr [\gamma_5 C_{nn'}(x) \gamma_5 C U_{mm'}^T(x) C] \\
& - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C S_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C U_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C U_{jm'}^T(x) C] \} , \tag{27}
\end{aligned}$$

$$\begin{aligned}
\Pi_{uss}^{10}(p) = & -\frac{i}{3} \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} \gamma_5 \gamma^\mu C C_{a'a}^T(-x) C \gamma^\nu \gamma_5 \\
& \{ 2Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{jj'}^T(x) C] Tr [\gamma_5 C_{nn'}(x) \gamma_5 C U_{mm'}^T(x) C] \\
& + 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C S_{jj'}^T(x) C] Tr [\gamma_5 C_{nn'}(x) \gamma_5 C S_{mm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C U_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C U_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C S_{jm'}^T(x) C] \\
& - 4Tr [\gamma_\mu U_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C S_{jm'}^T(x) C] \} , \tag{28}
\end{aligned}$$

$$\begin{aligned}
\Pi_{sss}^{10}(p) = & -i \varepsilon_{ila} \varepsilon_{ijk} \varepsilon_{lmn} \varepsilon_{i'l'a'} \varepsilon_{i'j'k'} \varepsilon_{l'm'n'} \int d^4x e^{ip \cdot x} \gamma_5 \gamma^\mu C C_{a'a}^T(-x) C \gamma^\nu \gamma_5 \\
& \{ 2Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{jj'}^T(x) C] Tr [\gamma_5 C_{nn'}(x) \gamma_5 C S_{mm'}^T(x) C] \\
& - 4Tr [\gamma_\mu S_{kk'}(x) \gamma_\nu C S_{mj'}^T(x) C \gamma_5 C_{nn'}(x) \gamma_5 C S_{jm'}^T(x) C] \} , \tag{29}
\end{aligned}$$

where the  $U_{ij}(x)$ ,  $S_{ij}(x)$  and  $C_{ij}(x)$  are the full  $u$ ,  $s$  and  $c$  quark propagators respectively,

$$\begin{aligned}
U_{ij}(x) &= \frac{i\delta_{ij} \not{x}}{2\pi^2 x^4} - \frac{\delta_{ij} \langle \bar{q}q \rangle}{12} - \frac{\delta_{ij} x^2 \langle \bar{q}g_s \sigma Gq \rangle}{192} - \frac{ig_s G_{\alpha\beta}^a t_{ij}^a (\not{x} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \not{x})}{32\pi^2 x^2} - \frac{1}{8} \langle \bar{q}_j \sigma^{\mu\nu} q_i \rangle \sigma_{\mu\nu} + \dots , \\
S_{ij}(x) &= \frac{i\delta_{ij} \not{x}}{2\pi^2 x^4} - \frac{\delta_{ij} m_s}{4\pi^2 x^2} - \frac{\delta_{ij} \langle \bar{s}s \rangle}{12} + \frac{i\delta_{ij} \not{x} m_s \langle \bar{s}s \rangle}{48} - \frac{\delta_{ij} x^2 \langle \bar{s}g_s \sigma Gs \rangle}{192} + \frac{i\delta_{ij} x^2 \not{x} m_s \langle \bar{s}g_s \sigma Gs \rangle}{1152} \\
&\quad - \frac{ig_s G_{\alpha\beta}^a t_{ij}^a (\not{x} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \not{x})}{32\pi^2 x^2} - \frac{1}{8} \langle \bar{s}_j \sigma^{\mu\nu} s_i \rangle \sigma_{\mu\nu} + \dots , \tag{30}
\end{aligned}$$

$$\begin{aligned}
C_{ij}(x) &= \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\delta_{ij}}{\not{k} - m_c} - \frac{g_s G_{\alpha\beta}^n t_{ij}^n \sigma^{\alpha\beta} (\not{k} + m_c) + (\not{k} + m_c) \sigma^{\alpha\beta}}{4(k^2 - m_c^2)^2} \right. \\
&\quad \left. - \frac{g_s^2 (t^a t^b)_{ij} G_{\alpha\beta}^a G_{\mu\nu}^b (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\nu\mu\beta})}{4(k^2 - m_c^2)^5} + \dots \right\} , \\
f^{\alpha\beta\mu\nu} &= (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c) \gamma^\mu (\not{k} + m_c) \gamma^\nu (\not{k} + m_c) , \tag{31}
\end{aligned}$$

and  $t^n = \frac{\lambda^n}{2}$ , the  $\lambda^n$  is the Gell-Mann matrix [21], then compute the integrals both in the coordinate and momentum spaces to obtain the correlation functions  $\Pi_{q_1 q_2 q_3}^{j_L j_H}(p)$ , therefore the QCD spectral densities  $\rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s)$  and  $\tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)$  at the quark level through the dispersion relation,

$$\frac{\text{Im} \Pi_{q_1 q_2 q_3}^{j_L j_H}(s)}{\pi} = \not{p} \rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s) + m_c \tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s) . \tag{32}$$

The explicit expressions of the  $\rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s)$  and  $\tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)$  are given in the appendix. In Eq.(30), we retain the term  $\langle \bar{q}_j \sigma_{\mu\nu} q_i \rangle$  ( $\langle \bar{s}_j \sigma_{\mu\nu} s_i \rangle$ ) comes from the Fierz re-arrangement of the  $\langle q_i \bar{q}_j \rangle$  ( $\langle s_i \bar{s}_j \rangle$ ) to absorb the gluons emitted from other quark lines to form  $\langle \bar{q}_j g_s G_{\alpha\beta}^a t_{mn}^a \sigma_{\mu\nu} q_i \rangle$  ( $\langle \bar{s}_j g_s G_{\alpha\beta}^a t_{mn}^a \sigma_{\mu\nu} s_i \rangle$ ) to extract the mixed condensate  $\langle \bar{q}g_s \sigma Gq \rangle$  ( $\langle \bar{s}g_s \sigma Gs \rangle$ ). A number of terms involving the mixed condensates  $\langle \bar{q}g_s \sigma Gq \rangle$  and  $\langle \bar{s}g_s \sigma Gs \rangle$  appear and play an important role in the QCD sum rules.

Once the analytical QCD spectral densities  $\rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s)$  and  $\tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)$  are obtained, we can take the quark-hadron duality below the continuum thresholds  $s_0$  and introduce the weight function  $\exp(-\frac{s}{T^2})$  to obtain the following QCD sum rules:

$$2M_{P,-} \lambda_P^{-2} \exp\left(-\frac{M_{P,-}^2}{T^2}\right) = \int_{4m_c^2}^{s_0} ds [\sqrt{s} \rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s) + m_c \tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)] \exp\left(-\frac{s}{T^2}\right) , \tag{33}$$

$$2M_{P,+} \lambda_P^{+2} \exp\left(-\frac{M_{P,+}^2}{T^2}\right) = \int_{4m_c^2}^{s_0} ds [\sqrt{s} \rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s) - m_c \tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)] \exp\left(-\frac{s}{T^2}\right) , \tag{34}$$

where we take into account the contributions of the terms  $D_0$ ,  $D_3$ ,  $D_5$ ,  $D_6$ ,  $D_8$ ,  $D_9$  and  $D_{10}$ ,

$$\begin{aligned}
D_0 &= \text{perturbative terms}, \\
D_3 &\propto \langle \bar{q}q \rangle, \langle \bar{s}s \rangle, \\
D_5 &\propto \langle \bar{q}g_s\sigma Gq \rangle, \langle \bar{s}g_s\sigma Gs \rangle, \\
D_6 &\propto \langle \bar{q}q \rangle^2, \langle \bar{q}q \rangle \langle \bar{s}s \rangle, \langle \bar{s}s \rangle^2, \\
D_8 &\propto \langle \bar{q}q \rangle \langle \bar{q}g_s\sigma Gq \rangle, \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle, \langle \bar{q}q \rangle \langle \bar{s}g_s\sigma Gs \rangle, \langle \bar{s}s \rangle \langle \bar{s}g_s\sigma Gs \rangle, \\
D_9 &\propto \langle \bar{q}q \rangle^3, \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2, \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle, \langle \bar{s}s \rangle^3, \\
D_{10} &\propto \langle \bar{q}g_s\sigma Gq \rangle^2, \langle \bar{q}g_s\sigma Gq \rangle \langle \bar{s}g_s\sigma Gs \rangle, \langle \bar{s}g_s\sigma Gs \rangle^2.
\end{aligned} \tag{35}$$

In this article, we carry out the operator product expansion to the vacuum condensates up to dimension-10, and assume vacuum saturation for the higher dimension vacuum condensates.

We differentiate Eqs.(33-34) with respect to  $\frac{1}{T^2}$ , then eliminate the pole residues  $\lambda_P^\pm$  and obtain the QCD sum rules for the masses of the pentaquark states,

$$M_{P,-}^2 = \frac{\int_{4m_c^2}^{s_0} ds s [\sqrt{s} \rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s) + m_c \tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)] \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0} ds [\sqrt{s} \rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s) + m_c \tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)] \exp\left(-\frac{s}{T^2}\right)}, \tag{36}$$

$$M_{P,+}^2 = \frac{\int_{4m_c^2}^{s_0} ds s [\sqrt{s} \rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s) - m_c \tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)] \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0} ds [\sqrt{s} \rho_{q_1 q_2 q_3}^{j_L j_H, 1}(s) - m_c \tilde{\rho}_{q_1 q_2 q_3}^{j_L j_H, 0}(s)] \exp\left(-\frac{s}{T^2}\right)}. \tag{37}$$

Once the masses  $M_{P,\pm}$  are obtained, we can take them as input parameters and obtain the pole residues from the QCD sum rules in Eqs.(33-34). The gluon condensates are associated with large numerical denominators, their contributions to total QCD spectral densities are less (or much less) than the contributions of the dimension 10 vacuum condensates  $D_{10}$  for the pentaquark currents with the axialvector diquark constituents [7, 8]. We obtain the masses through fractions, see Eqs.(36-37), the effects of the gluon condensates can be safely absorbed into the pole residues  $\lambda_P^\pm$  and tiny effects on the masses can be safely neglected.

### 3 Numerical results and discussions

The vacuum condensates are taken to be the standard values  $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$ ,  $\langle \bar{q}g_s\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ ,  $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$ ,  $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$  at the energy scale  $\mu = 1 \text{ GeV}$  [20, 21]. The quark condensates and mixed quark condensates evolve with the renormalization group equation,  $\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{4}{9}}$ ,  $\langle \bar{s}s \rangle(\mu) = \langle \bar{s}s \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{4}{9}}$ ,  $\langle \bar{q}g_s\sigma Gq \rangle(\mu) = \langle \bar{q}g_s\sigma Gq \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{2}{27}}$  and  $\langle \bar{s}g_s\sigma Gs \rangle(\mu) = \langle \bar{s}g_s\sigma Gs \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{2}{27}}$ .

In the article, we take the  $\overline{MS}$  masses  $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$  and  $m_s(\mu = 2 \text{ GeV}) = (0.095 \pm 0.005) \text{ GeV}$  from the Particle Data Group [22], and take into account the energy-scale dependence of the  $\overline{MS}$  masses from the renormalization group equation,

$$\begin{aligned}
m_c(\mu) &= m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{25}}, \\
m_s(\mu) &= m_s(2 \text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{4}{9}}, \\
\alpha_s(\mu) &= \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right],
\end{aligned} \tag{38}$$



where  $t = \log \frac{\mu^2}{\Lambda^2}$ ,  $b_0 = \frac{33-2n_f}{12\pi}$ ,  $b_1 = \frac{153-19n_f}{24\pi^2}$ ,  $b_2 = \frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$ ,  $\Lambda = 213 \text{ MeV}$ ,  $296 \text{ MeV}$  and  $339 \text{ MeV}$  for the flavors  $n_f = 5, 4$  and  $3$ , respectively [22]. Furthermore, we set the small masses of the  $u$  and  $d$  quarks equal zero,  $m_u = m_d = 0$ .

In Ref.[19], we study the  $J^P = \frac{1}{2}^\pm$  and  $\frac{3}{2}^\pm$  heavy, doubly-heavy and triply-heavy baryon states with the QCD sum rules in a systematic way by subtracting the contributions from the corresponding  $J^P = \frac{1}{2}^\mp$  and  $\frac{3}{2}^\mp$  heavy, doubly-heavy and triply-heavy baryon states, the continuum threshold parameters  $\sqrt{s_0} = M_{\text{gr}} + (0.6 - 0.8) \text{ GeV}$  work well, where subscript gr denotes the ground state baryons. The pentaquark states are another type baryon states according to the fractional spins, in Ref.[7], we take the continuum threshold parameters as  $\sqrt{s_0} = M_{P_c(4380/4450)} + (0.6 - 0.8) \text{ GeV}$ , which also works well. In this article, we take the continuum threshold parameters  $\sqrt{s_0} = M_P + (0.6 - 0.8) \text{ GeV}$  as an additional constraint.

The hidden charm (or bottom) five-quark systems  $q_1 q_2 q_3 Q \bar{Q}$  could be described by a double-well potential, just like the double heavy four-quark systems  $q_1 \bar{q}_2 Q \bar{Q}$  [23]. We introduce the color indexes  $i, j$  and  $k$  firstly. In the five-quark system  $q_1 q_2 q_3 Q \bar{Q}$ , the light quarks  $q_1$  and  $q_2$  combine together to form a light diquark  $\mathcal{D}_{q_1 q_2}^i$  in color antitriplet,

$$q_1 + q_2 \rightarrow \mathcal{D}_{q_1 q_2}^i, \quad (39)$$

the  $Q$ -quark serves as a static well potential and combines with the light quark  $q_3$  to form a heavy diquark  $\mathcal{D}_{q_3 Q}^j$  in color antitriplet,

$$q_3 + Q \rightarrow \mathcal{D}_{q_3 Q}^j, \quad (40)$$

the  $\bar{Q}$ -quark serves as another static well potential and combines with the light diquark  $\mathcal{D}_{q_1 q_2}^i$  to form a heavy triquark in color triplet,

$$\bar{Q}^k + \mathcal{D}_{q_1 q_2}^i \rightarrow \varepsilon^{jki} \bar{Q}^k \mathcal{D}_{q_1 q_2}^i. \quad (41)$$

Then the heavy diquark  $\mathcal{D}_{q_3 Q}^j$  combines with the heavy triquark  $\varepsilon^{jki} \bar{Q}^k \mathcal{D}_{q_1 q_2}^i$  to form a pentaquark state in color singlet,

$$\varepsilon^{jki} \bar{Q}^k \mathcal{D}_{q_1 q_2}^i + \mathcal{D}_{q_3 Q}^j \rightarrow \varepsilon^{ijk} \mathcal{D}_{q_1 q_2}^i \mathcal{D}_{q_3 Q}^j \bar{Q}^k. \quad (42)$$

The interpolating currents in Eqs.(7-14) can also be understood in this way.

Now we can see that the double heavy five-quark system is characterized by the effective heavy quark masses  $\mathbb{M}_Q$  and the virtuality  $V = \sqrt{M_P^2 - (2\mathbb{M}_Q)^2}$ , just like the double heavy four-quark systems  $q_1 \bar{q}_2 Q \bar{Q}$  [23]. The QCD sum rules have three typical energy scales  $\mu^2$ ,  $T^2$ ,  $V^2$ , we set the energy scales to be  $\mu^2 = V^2 = \mathcal{O}(T^2)$ , and obtain energy scale formula,

$$\mu = \sqrt{M_P^2 - (2\mathbb{M}_Q)^2}, \quad (43)$$

to determine the energy scales of the QCD spectral densities [7, 8]. In previous work [7], we take the value  $\mathbb{M}_c = 1.8 \text{ GeV}$  determined in the double heavy four-quark systems [23] and obtain the values  $\mu = 2.5 \text{ GeV}$  and  $\mu = 2.6 \text{ GeV}$  for the hidden-charm pentaquark states  $P_c(4380)$  and  $P_c(4450)$ , respectively. The energy scale formula works well.

We can rewrite Eq.(43) into the following form,

$$M_P^2 = 4\mathbb{M}_c^2 + \mu^2, \quad (44)$$

which indicates that the masses increase (or decrease) with increase (or decrease) of the energy scales,

$$\begin{aligned} \mu \uparrow & \quad M_P \uparrow, \\ \mu \downarrow & \quad M_P \downarrow. \end{aligned} \quad (45)$$

On the other hand, the calculations based on the QCD sum rules in Eqs.(36-37) indicate that the masses decrease (or increase) with increase (or decrease) of the energy scales,

$$\begin{aligned}\mu \uparrow & \quad M_P \downarrow, \\ \mu \downarrow & \quad M_P \uparrow.\end{aligned}\tag{46}$$

We can search for a compromise and obtain the optimal energy scales  $\mu$  and masses  $M_P$ .

In the present QCD sum rules, we choose the Borel parameters  $T^2$  and continuum threshold parameters  $s_0$  to satisfy the following four criteria:

- Pole dominance at the phenomenological side;
- Convergence of the operator product expansion;
- Appearance of the Borel platforms;
- Satisfying the energy scale formula.

Now we search for the optimal Borel parameters  $T^2$  and continuum threshold parameters  $s_0$  according to the four criteria. The resulting Borel parameters  $T^2$ , continuum threshold parameters  $s_0$ , pole contributions, and contributions of the vacuum condensates of dimension 9 and 10 are shown explicitly in Table 1. From the table, we can see that the first two criteria of the QCD sum rules are satisfied, and we expect to make reasonable predictions. In Table 2, we present the contributions of different terms in the operator product expansion with the central values of the input parameters. In calculations, we observe that the main contributions come from the terms  $D_0$ ,  $D_3$  and  $D_6$ , see Table 2, the operator product expansion is well convergent. We should admit that the convergent behavior of the operator product expansion is not so good as that in the conventional case, where the  $D_0$  dominates the QCD sum rules, but we can find a comparatively reasonable work window to extract the hadronic information [24].

We take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the  $\frac{1}{2}^\pm$  hidden-charm pentaquark states, which are shown in Figs.1-2 and Table 3. From Figs.1-2 and Table 3, we can see that the last two criteria are also satisfied. In Table 3, we also present the corresponding thresholds of the  $J/\psi B_{10}$  and  $J/\psi B_8$ , where the  $B_8$  and  $B_{10}$  denote the octet and decuplet baryons with the constituents  $q_1 q_2 q_3$ , respectively. From the table, we can see that the decays to the  $J/\psi B_{10}$ ,

$$P_{q_1 q_2 q_3}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right), P_{q_1 q_2 q_3}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right) \rightarrow J/\psi B_{10},\tag{47}$$

for example,

$$\begin{aligned}P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right), P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right) & \rightarrow J/\psi \Delta^{++}, \\ P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right), P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right) & \rightarrow J/\psi \Sigma^{*+}, \\ P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right), P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right) & \rightarrow J/\psi \Xi^{*0}, \\ P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right), P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right) & \rightarrow J/\psi \Omega^-, \end{aligned}\tag{48}$$

can (or maybe) take place, but the decay widths are rather small due to the small available phase-spaces; on the other hand, the decays,

$$P_{q_1 q_2 q_3}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right), P_{q_1 q_2 q_3}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right) \rightarrow J/\psi B_8,\tag{49}$$

$$P_{q_1 q_2 q_3}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right) \rightarrow J/\psi B_{10}, J/\psi B_8,\tag{50}$$

for example,

$$\begin{aligned}
P_{uud}^{11\frac{1}{2}}\left(\frac{1}{2}^{\pm}\right), P_{uud}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right) &\rightarrow J/\psi p, \\
P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{\pm}\right), P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right) &\rightarrow J/\psi \Sigma^{+}, \\
P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{\pm}\right), P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right) &\rightarrow J/\psi \Xi^{0}, \\
P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Delta^{++}, \\
P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Sigma^{*+}, \\
P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Xi^{*0}, \\
P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Omega^{-},
\end{aligned} \tag{51}$$

can take place more easily, the decay widths are larger due to the larger available phase-spaces; furthermore, the decays

$$P_{q_1 q_2 q_3}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) \rightarrow J/\psi B_8, J/\psi B_{10}, \tag{52}$$

for example,

$$\begin{aligned}
P_{uud}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi p, \\
P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Sigma^{+}, \\
P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Xi^{0}, \\
P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Delta^{++}, \\
P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Sigma^{*+}, \\
P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Xi^{*0}, \\
P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right) &\rightarrow J/\psi \Omega^{-},
\end{aligned} \tag{53}$$

can take place fluently, the decay widths are rather large due to the large available phase-spaces. We can search for the pentaquark states in the  $J/\psi B_8$  and  $J/\psi B_{10}$  mass spectrum in the decays of the bottom baryons to the final states  $J/\psi B_8$  and  $J/\psi B_{10}$  associated with the light vector mesons or pseudoscalar mesons [4, 6, 9], for example,

$$\begin{aligned}
\Omega_b^{-} &\rightarrow P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{\pm}\right) K^{-} \rightarrow J/\psi \Xi^{*0} K^{-}, \\
\Omega_b^{-} &\rightarrow P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^{\pm}\right) \phi \rightarrow J/\psi \Omega^{-} \phi.
\end{aligned} \tag{54}$$

	$T^2(\text{GeV}^2)$	$\sqrt{s_0}(\text{GeV})$	pole	$D_9$	$D_{10}$
$P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.2 – 3.6	$5.1 \pm 0.1$	(38 – 60)%	(16 – 22)%	$\sim 1\%$
$P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.3 – 3.7	$5.2 \pm 0.1$	(39 – 60)%	(10 – 15)%	$\leq 1\%$
$P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.4 – 3.8	$5.3 \pm 0.1$	(40 – 61)%	(7 – 10)%	$< 1\%$
$P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.5 – 3.9	$5.4 \pm 0.1$	(42 – 62)%	(5 – 7)%	$< 1\%$
$P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.3 – 3.7	$5.1 \pm 0.1$	(39 – 60)%	(10 – 13)%	(3 – 4)%
$P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.4 – 3.8	$5.2 \pm 0.1$	(41 – 61)%	(6 – 9)%	(2 – 3)%
$P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.5 – 3.9	$5.3 \pm 0.1$	(42 – 62)%	(4 – 6)%	(1 – 2)%
$P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^-\right)$	3.6 – 4.0	$5.4 \pm 0.1$	(43 – 62)%	(3 – 4)%	$\sim 1\%$
$P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.3 – 3.7	$5.3 \pm 0.1$	(37 – 59)%	(15 – 21)%	$\leq 1\%$
$P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.4 – 3.8	$5.4 \pm 0.1$	(38 – 59)%	(10 – 14)%	$< 1\%$
$P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.5 – 3.9	$5.5 \pm 0.1$	(39 – 60)%	(7 – 10)%	$< 1\%$
$P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.6 – 4.0	$5.6 \pm 0.1$	(40 – 60)%	(5 – 7)%	$< 1\%$
$P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.3 – 3.7	$5.8 \pm 0.1$	(60 – 78)%	–(3 – 5)%	$< 1\%$
$P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.4 – 3.8	$5.9 \pm 0.1$	(61 – 79)%	–(2 – 4)%	$< 1\%$
$P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.5 – 3.9	$6.0 \pm 0.1$	(62 – 79)%	–(1 – 2)%	$< 1\%$
$P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$	3.6 – 4.0	$6.1 \pm 0.1$	(63 – 80)%	$\sim -1\%$	$< 1\%$

Table 1: The Borel parameters, continuum threshold parameters, pole contributions, contributions of the vacuum condensates of dimension 9 and dimension 10.

In Ref.[10], the authors study the non-strange and strange pentaquarks with hidden-charm in the diquark-diquark-antiquark model by considering the simple spin-spin interactions, and evaluate the masses, where the scalar and axialvector diquarks are chosen. The predicted masses are different from ours, see Refs.[7, 8] and present work, the differences originate partly from the fact that in Ref.[10] the  $P_c(4450)$  is taken as the  $P_{uud}^{11\frac{1}{2}}(\frac{5}{2}^-)$  pentaquark state, while in Refs.[7, 8] the  $P_c(4450)$  is taken as the  $P_{uud}^{11\frac{3}{2}}(\frac{5}{2}^+)$  pentaquark state.

## 4 Conclusion

In this article, we choose the axialvector-diquark-axialvector-diquark-antiquark type and axialvector-diquark-scalar-diquark-antiquark type pentaquark configurations, construct both the axialvector-diquark-axialvector-diquark-antiquark type and axialvector-diquark-scalar-diquark-antiquark type interpolating currents, then calculate the contributions of the vacuum condensates up to dimension-10 in the operator product expansion, and study the masses and pole residues of the  $J^P = \frac{1}{2}^\pm$  hidden-charm pentaquark states with the QCD sum rules in a systematic way. In calculations, we use the formula  $\mu = \sqrt{M_P^2 - (2M_c)^2}$  to determine the energy scales of the QCD spectral densities, which works well in our previous work. We take into account the  $SU(3)$  breaking effects of the light quarks, and obtain the masses of the hidden charm pentaquark states with the strangeness  $S = 0, -1, -2, -3$ , which can be confronted with the experimental data in the future. We can

Pentaquark states	Contributions
$P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_0 \gg D_6 > D_3 \gg  D_5  \approx  D_8  \approx D_9 \gg D_{10}$
$P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_0 \gg D_6 > D_3 \gg  D_5  \approx  D_8  > D_9 \gg D_{10}$
$P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_0 \gg D_6 > D_3 \gg  D_5  \approx  D_8  > D_9 \gg D_{10}$
$P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_0 \gg D_6 > D_3 >  D_5  >  D_8  > D_9 \gg D_{10}$
$P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_6 > D_3 > D_0 >  D_8  \gg  D_5  > D_9 \gg D_{10}$
$P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_0 \approx D_3 > D_6 \gg  D_8  >  D_5  \gg D_9 \gg D_{10}$
$P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_0 \gg D_3 > D_6 \gg  D_8  >  D_5  \gg D_9 \gg D_{10}$
$P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	$D_0 \gg D_3 \gg D_6 \gg  D_8  >  D_5  \gg D_9 \gg D_{10}$
$P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_6 > D_3 \gg  D_5  \approx  D_8  \approx D_9 \gg D_{10}$
$P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_6 > D_3 \gg  D_5  \approx  D_8  > D_9 \gg D_{10}$
$P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_6 > D_3 \gg  D_5  \approx  D_8  > D_9 \gg D_{10}$
$P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_6 > D_3 \gg  D_5  >  D_8  > D_9 \gg D_{10}$
$P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_3 \gg  D_6  >  D_5  \gg D_8 >  D_9  \gg  D_{10} $
$P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_3 \gg  D_6  >  D_5  \gg D_8 >  D_9  \gg  D_{10} $
$P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_3 \gg  D_6  >  D_5  \gg D_8 \gg  D_9  \gg  D_{10} $
$P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	$D_0 \gg D_3 \gg  D_6  >  D_5  \gg  D_9  > D_8 \gg  D_{10} $

Table 2: The contributions of different terms in the operator product expansion with the central values of the input parameters.

	$\mu(\text{GeV})$	$M_P(\text{GeV})$	$\lambda_P(\text{GeV}^6)$	$M_{B_{10}J/\psi(B_8J/\psi)}(\text{GeV})$
$P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	2.5	$4.35 \pm 0.15$	$(3.72 \pm 0.76) \times 10^{-3}$	4.33 (4.04)
$P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	2.6	$4.47 \pm 0.15$	$(4.50 \pm 0.85) \times 10^{-3}$	4.48 (4.29)
$P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	2.8	$4.58 \pm 0.14$	$(5.43 \pm 0.96) \times 10^{-3}$	4.63 (4.41)
$P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	3.0	$4.68 \pm 0.13$	$(6.47 \pm 1.10) \times 10^{-3}$	4.77
$P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	2.5	$4.42 \pm 0.12$	$(4.14 \pm 0.70) \times 10^{-3}$	4.33 (4.04)
$P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	2.7	$4.51 \pm 0.11$	$(4.97 \pm 0.79) \times 10^{-3}$	4.48 (4.29)
$P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	2.9	$4.60 \pm 0.11$	$(5.87 \pm 0.89) \times 10^{-3}$	4.63 (4.41)
$P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$	3.0	$4.71 \pm 0.11$	$(6.84 \pm 1.00) \times 10^{-3}$	4.77
$P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	2.8	$4.56 \pm 0.15$	$(1.97 \pm 0.40) \times 10^{-3}$	4.33 (4.04)
$P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	3.0	$4.67 \pm 0.14$	$(2.42 \pm 0.47) \times 10^{-3}$	4.48 (4.29)
$P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	3.1	$4.78 \pm 0.13$	$(2.88 \pm 0.53) \times 10^{-3}$	4.63 (4.41)
$P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	3.3	$4.89 \pm 0.13$	$(3.44 \pm 0.61) \times 10^{-3}$	4.77
$P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	3.6	$5.12 \pm 0.08$	$(8.01 \pm 0.92) \times 10^{-3}$	4.33 (4.04)
$P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	3.7	$5.19 \pm 0.08$	$(8.92 \pm 1.05) \times 10^{-3}$	4.48 (4.29)
$P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	3.8	$5.26 \pm 0.08$	$(9.93 \pm 1.21) \times 10^{-3}$	4.63 (4.41)
$P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^{+}\right)$	4.0	$5.40 \pm 0.08$	$(12.17 \pm 1.28) \times 10^{-3}$	4.77

Table 3: The energy scales, masses and pole residues of the pentaquark states, where the  $B_{10}$  and  $B_8$  denote the decuplet and octet baryons with the quark constituents  $q_1 q_2 q_3$  respectively. We take the isospin limit, the pentaquark states  $P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{udd}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$  and  $P_{ddd}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$  have degenerate masses. Other states are implied.

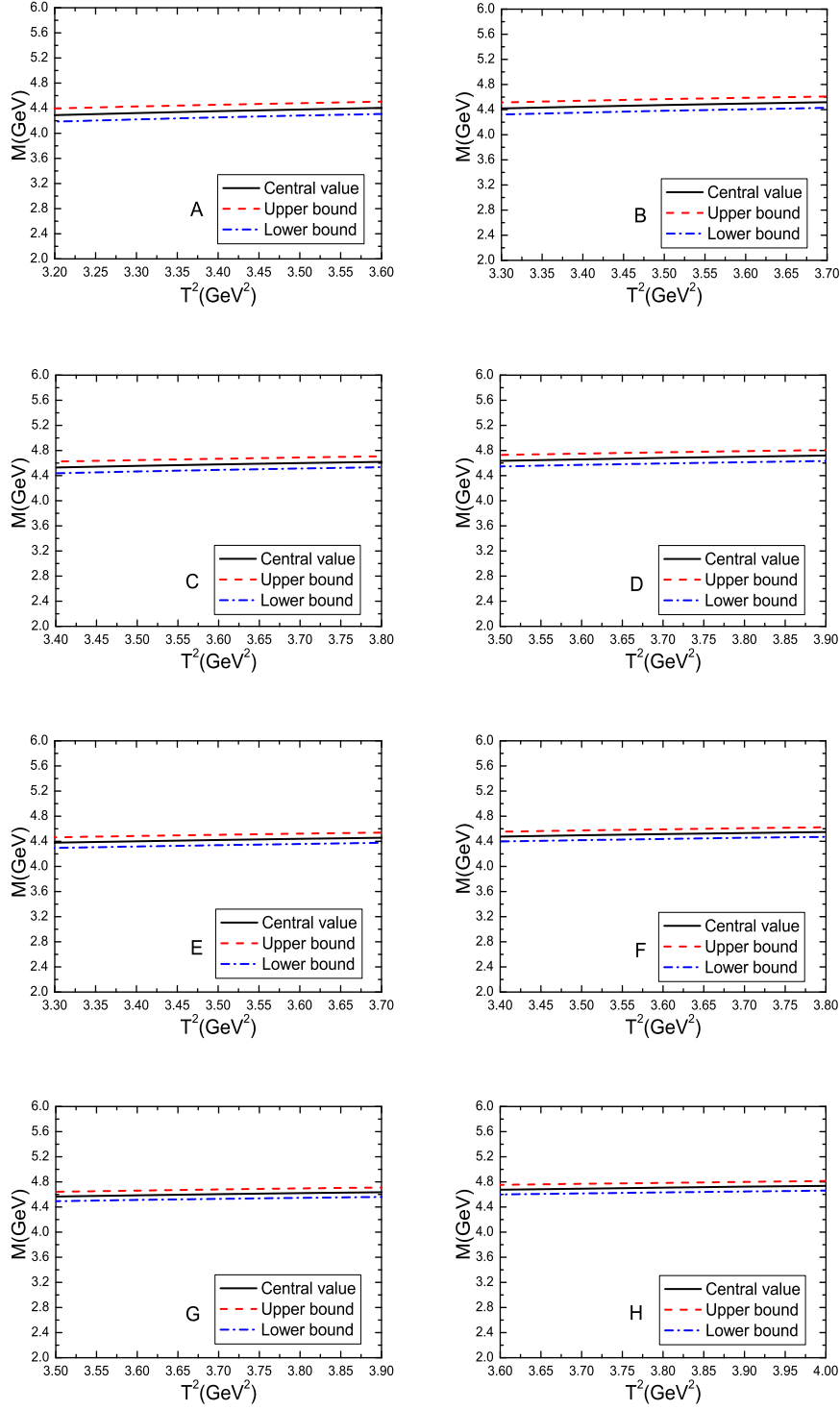


Figure 1: The masses of the pentaquark states with variations of the Borel parameters  $T^2$ , where the  $A, B, C, D, E, F, G$  and  $H$  denote the pentaquark states  $P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$ ,  $P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$  and  $P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^{-}\right)$  with negative parity respectively.

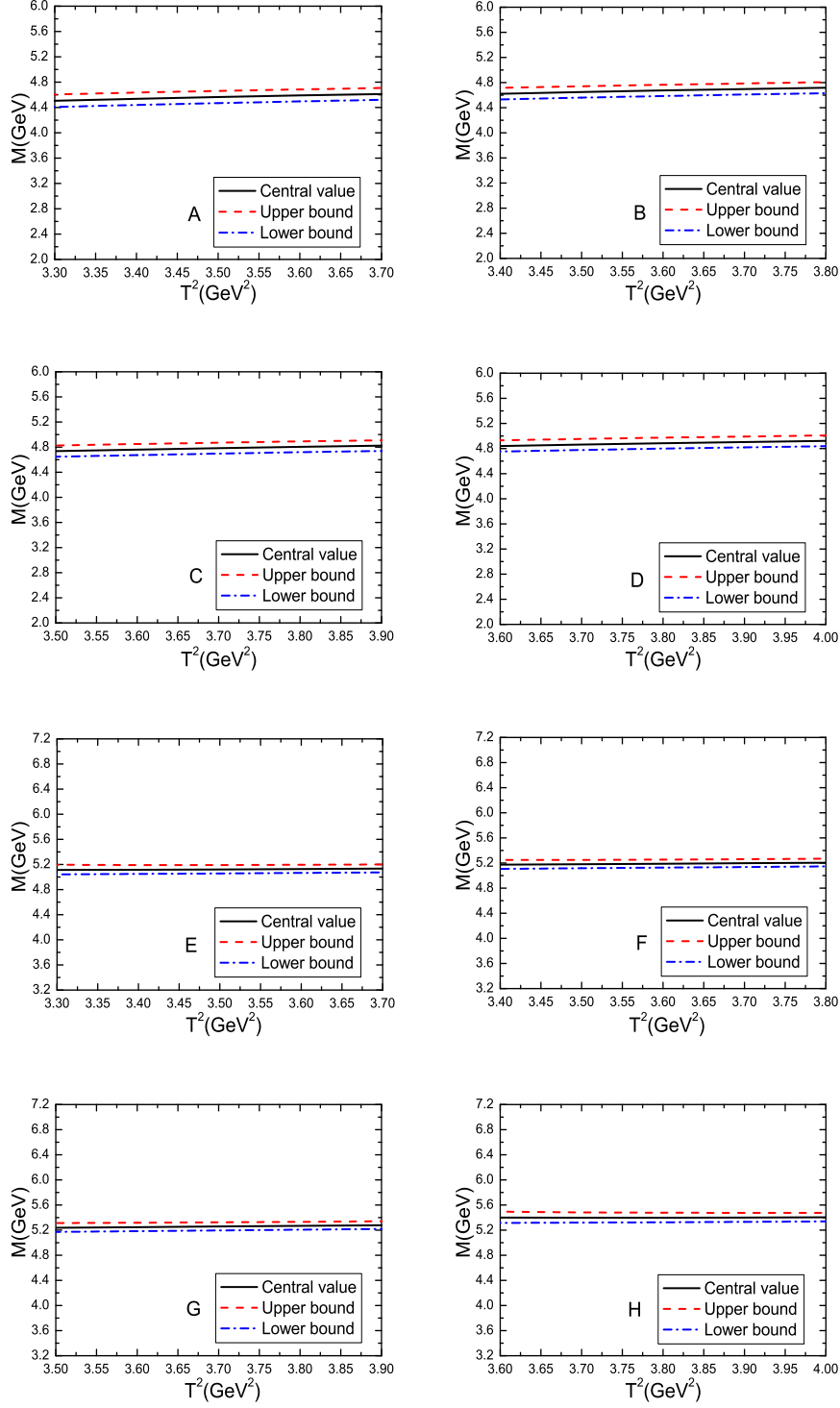


Figure 2: The masses of the pentaquark states with variations of the Borel parameters  $T^2$ , where the  $A, B, C, D, E, F, G$  and  $H$  denote the pentaquark states  $P_{uuu}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$ ,  $P_{uus}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$ ,  $P_{uss}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$ ,  $P_{sss}^{11\frac{1}{2}}\left(\frac{1}{2}^+\right)$ ,  $P_{uuu}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$ ,  $P_{uus}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$ ,  $P_{uss}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$  and  $P_{sss}^{10\frac{1}{2}}\left(\frac{1}{2}^+\right)$  with positive parity respectively.



search for those pentaquark states in the decays of the bottom baryons to the  $J/\psi$  plus octet (decuplet) baryons plus pseudoscalar (vector) mesons, or take the pole residues as basic input parameters to study relevant processes of the pentaquark states with the three-point QCD sum rules.

## Acknowledgements

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## Appendix

The QCD spectral densities  $\rho_{sss}^{11,1}(s)$ ,  $\tilde{\rho}_{sss}^{11,0}(s)$ ,  $\rho_{uss}^{11,1}(s)$ ,  $\tilde{\rho}_{uss}^{11,0}(s)$ ,  $\rho_{uus}^{11,1}(s)$ ,  $\tilde{\rho}_{uus}^{11,0}(s)$ ,  $\rho_{uuu}^{11,1}(s)$ ,  $\tilde{\rho}_{uuu}^{11,0}(s)$ ,  $\rho_{sss}^{10,1}(s)$ ,  $\tilde{\rho}_{sss}^{10,0}(s)$ ,  $\rho_{uss}^{10,1}(s)$ ,  $\tilde{\rho}_{uss}^{10,0}(s)$ ,  $\rho_{uus}^{10,1}(s)$ ,  $\tilde{\rho}_{uus}^{10,0}(s)$ ,  $\rho_{uuu}^{10,1}(s)$  and  $\tilde{\rho}_{uuu}^{10,0}(s)$  of the pentaquark states,

$$\begin{aligned}
\rho_{sss}^{11,1}(s) = & \frac{1}{61440\pi^8} \int dydz yz(1-y-z)^4 (s-\bar{m}_c^2)^4 (8s-3\bar{m}_c^2) \\
& + \frac{m_s m_c}{12288\pi^8} \int dydz (y+z)(1-y-z)^3 (s-\bar{m}_c^2)^4 \\
& - \frac{m_c \langle \bar{s}s \rangle}{768\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\bar{m}_c^2)^3 \\
& + \frac{m_s \langle \bar{s}s \rangle}{128\pi^6} \int dydz yz(1-y-z)^2 (s-\bar{m}_c^2)^2 (2s-\bar{m}_c^2) \\
& + \frac{3m_c \langle \bar{s}g_s \sigma G s \rangle}{1024\pi^6} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2)^2 \\
& + \frac{m_s \langle \bar{s}g_s \sigma G s \rangle}{512\pi^6} \int dydz yz(1-y-z) (s-\bar{m}_c^2) (5s-3\bar{m}_c^2) \\
& - \frac{m_s \langle \bar{s}g_s \sigma G s \rangle}{1024\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\bar{m}_c^2) (5s-3\bar{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle^2}{48\pi^4} \int dydz yz(1-y-z) (s-\bar{m}_c^2) (5s-3\bar{m}_c^2) \\
& + \frac{5m_s m_c \langle \bar{s}s \rangle^2}{48\pi^4} \int dydz (y+z) (s-\bar{m}_c^2) \\
& - \frac{5\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{192\pi^4} \int dydz yz (4s-3\bar{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{192\pi^4} \int dydz (y+z)(1-y-z) (4s-3\bar{m}_c^2) \\
& - \frac{121m_s m_c \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{2304\pi^4} \int dy + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{48\pi^4} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) \\
& - \frac{m_c \langle \bar{s}s \rangle^3}{18\pi^2} \int dy + \frac{m_s \langle \bar{s}s \rangle^3}{12\pi^2} \int dy y(1-y) \left[ 1 + \frac{s}{3} \delta(s-\tilde{m}_c^2) \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{3\langle\bar{s}g_s\sigma Gs\rangle^2}{512\pi^4}\int dydz(y+z)\left[1+\frac{s}{3}\delta(s-\bar{m}_c^2)\right] \\
& +\frac{3\langle\bar{s}g_s\sigma Gs\rangle^2}{256\pi^4}\int dy y(1-y)\left[1+\frac{s}{3}\delta(s-\tilde{m}_c^2)\right] \\
& -\frac{m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{192\pi^4}\int dy\left(\frac{1-y}{y}+\frac{y}{1-y}\right)\delta(s-\tilde{m}_c^2) \\
& +\frac{m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{72\pi^4}\int dy y(1-y)\left(1+\frac{s}{2M^2}\right)\delta(s-\tilde{m}_c^2) \\
& -\frac{m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{2304\pi^4}\int dy\left(1+\frac{s}{2M^2}\right)\delta(s-\tilde{m}_c^2), \tag{55}
\end{aligned}$$

$$\begin{aligned}
\hat{\rho}_{sss}^{11,0}(s) = & \frac{1}{122880\pi^8}\int dydz(y+z)(1-y-z)^4(s-\bar{m}_c^2)^4(7s-2\bar{m}_c^2) \\
& +\frac{m_sm_c}{6144\pi^8}\int dydz(1-y-z)^3(s-\bar{m}_c^2)^4 \\
& -\frac{m_c\langle\bar{s}s\rangle}{384\pi^6}\int dydz(1-y-z)^2(s-\bar{m}_c^2)^3 \\
& +\frac{m_s\langle\bar{s}s\rangle}{768\pi^6}\int dydz(y+z)(1-y-z)^2(s-\bar{m}_c^2)^2(5s-2\bar{m}_c^2) \\
& +\frac{3m_c\langle\bar{s}g_s\sigma Gs\rangle}{512\pi^6}\int dydz(1-y-z)(s-\bar{m}_c^2)^2 \\
& -\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{256\pi^6}\int dydz(1-y-z)^2(s-\bar{m}_c^2)(2s-\bar{m}_c^2) \\
& +\frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{512\pi^6}\int dydz(y+z)(1-y-z)(s-\bar{m}_c^2)(2s-\bar{m}_c^2) \\
& +\frac{\langle\bar{s}s\rangle^2}{48\pi^4}\int dydz(y+z)(1-y-z)(s-\bar{m}_c^2)(2s-\bar{m}_c^2) \\
& +\frac{5m_sm_c\langle\bar{s}s\rangle^2}{24\pi^4}\int dydz(s-\bar{m}_c^2) \\
& -\frac{5\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{384\pi^4}\int dydz(y+z)(3s-2\bar{m}_c^2) \\
& +\frac{\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{96\pi^4}\int dydz(1-y-z)(3s-2\bar{m}_c^2) \\
& -\frac{121m_sm_c\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{1152\pi^4}\int dy+\frac{m_sm_c\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{48\pi^4}\int dydz\left(\frac{1}{y}+\frac{1}{z}\right) \\
& -\frac{m_c\langle\bar{s}s\rangle^3}{9\pi^2}\int dy+\frac{m_s\langle\bar{s}s\rangle^3}{36\pi^2}\int dy\left[1+\frac{s}{2}\delta(s-\tilde{m}_c^2)\right] \\
& -\frac{\langle\bar{s}g_s\sigma Gs\rangle^2}{256\pi^4}\int dydz\left[1+\frac{s}{2}\delta(s-\bar{m}_c^2)\right] \\
& -\frac{m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{192\pi^4}\int dy\left(\frac{1}{y}+\frac{1}{1-y}\right)\delta(s-\tilde{m}_c^2) \\
& +\frac{31m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{2304\pi^4}\int dy\left(1+\frac{s}{M^2}\right)\delta(s-\tilde{m}_c^2), \tag{56}
\end{aligned}$$

$$\begin{aligned}
\rho_{uss}^{11,1}(s) = & \frac{1}{61440\pi^8} \int dydz yz(1-y-z)^4 (s - \overline{m}_c^2)^4 (8s - 3\overline{m}_c^2) \\
& + \frac{m_s m_c}{18432\pi^8} \int dydz (y+z)(1-y-z)^3 (s - \overline{m}_c^2)^4 \\
& - \frac{m_c [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^2 (s - \overline{m}_c^2)^3 \\
& + \frac{m_s [2\langle \bar{s}s \rangle - \langle \bar{q}q \rangle]}{192\pi^6} \int dydz yz(1-y-z)^2 (s - \overline{m}_c^2)^2 (2s - \overline{m}_c^2) \\
& + \frac{m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{1024\pi^6} \int dydz (y+z)(1-y-z) (s - \overline{m}_c^2)^2 \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{384\pi^6} \int dydz yz(1-y-z) (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{1536\pi^6} \int dydz yz(1-y-z) (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& - \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{3072\pi^6} \int dydz (y+z)(1-y-z)^2 (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{144\pi^4} \int dydz yz(1-y-z) (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [11\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{144\pi^4} \int dydz (y+z) (s - \overline{m}_c^2) \\
& - \frac{5 [\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{576\pi^4} \int dydz yz (4s - 3\overline{m}_c^2) \\
& + \frac{\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle}{576\pi^4} \int dydz (y+z)(1-y-z) (4s - 3\overline{m}_c^2) \\
& + \frac{m_s m_c [\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{144\pi^4} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) \\
& - \frac{m_s m_c [34\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + 33\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle - 5\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{2304\pi^4} \int dy \\
& - \frac{m_c \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{18\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{18\pi^2} \int dy y(1-y) \left[ 1 + \frac{s}{3} \delta(s - \tilde{m}_c^2) \right] \\
& - \frac{\langle \bar{s}g_s \sigma Gs \rangle [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{512\pi^4} \int dydz (y+z) \left[ 1 + \frac{s}{3} \delta(s - \tilde{m}_c^2) \right] \\
& + \frac{\langle \bar{s}g_s \sigma Gs \rangle [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{256\pi^4} \int dy y(1-y) \left[ 1 + \frac{s}{3} \delta(s - \tilde{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle \langle \bar{s}g_s \sigma Gs \rangle}{288\pi^4} \int dy \left( \frac{1-y}{y} + \frac{y}{1-y} \right) \delta(s - \tilde{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [17\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \left( 1 + \frac{s}{2M^2} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{6912\pi^4} \int dy \left( 1 + \frac{s}{2M^2} \right) \delta(s - \tilde{m}_c^2) , \tag{57}
\end{aligned}$$

$$\begin{aligned}
\tilde{\rho}_{uss}^{11,0}(s) = & \frac{1}{122880\pi^8} \int dydz (y+z)(1-y-z)^4 (s-\bar{m}_c^2)^4 (7s-2\bar{m}_c^2) \\
& + \frac{m_s m_c}{9216\pi^8} \int dydz (1-y-z)^3 (s-\bar{m}_c^2)^4 \\
& - \frac{m_c [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{1152\pi^6} \int dydz (1-y-z)^2 (s-\bar{m}_c^2)^3 \\
& + \frac{m_s [2\langle \bar{s}s \rangle - \langle \bar{q}q \rangle]}{1152\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\bar{m}_c^2)^2 (5s-2\bar{m}_c^2) \\
& + \frac{m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{512\pi^6} \int dydz (1-y-z) (s-\bar{m}_c^2)^2 \\
& - \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{768\pi^6} \int dydz (1-y-z)^2 (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{384\pi^6} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{1536\pi^6} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{144\pi^4} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [11\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{72\pi^4} \int dydz (s-\bar{m}_c^2) \\
& - \frac{5 [\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{1152\pi^4} \int dydz (y+z) (3s-2\bar{m}_c^2) \\
& + \frac{[\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{288\pi^4} \int dydz (1-y-z) (3s-2\bar{m}_c^2) \\
& + \frac{m_s m_c [\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{144\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) \\
& - \frac{m_s m_c [34\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + 33\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle - 5\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{864\pi^4} \int dy \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{1152\pi^4} \int dy \\
& - \frac{m_c \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{9\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{54\pi^2} \int dy \left[ 1 + \frac{s}{2} \delta(s-\tilde{m}_c^2) \right] \\
& - \frac{\langle \bar{s}g_s \sigma Gs \rangle [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{768\pi^4} \int dydz \left[ 1 + \frac{s}{2} \delta(s-\tilde{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle \langle \bar{s}g_s \sigma Gs \rangle}{288\pi^4} \int dy \left( \frac{1}{y} + \frac{1}{1-y} \right) \delta(s-\tilde{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [17\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s-\tilde{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{6912\pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s-\tilde{m}_c^2), \tag{58}
\end{aligned}$$

$$\begin{aligned}
\rho_{uus}^{11,1}(s) = & \frac{1}{61440\pi^8} \int dydz yz(1-y-z)^4 (s - \overline{m}_c^2)^4 (8s - 3\overline{m}_c^2) \\
& + \frac{m_s m_c}{36864\pi^8} \int dydz (y+z)(1-y-z)^3 (s - \overline{m}_c^2)^4 \\
& - \frac{m_c [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^2 (s - \overline{m}_c^2)^3 \\
& + \frac{m_s [3\langle \bar{s}s \rangle - 2\langle \bar{q}q \rangle]}{384\pi^6} \int dydz yz(1-y-z)^2 (s - \overline{m}_c^2)^2 (2s - \overline{m}_c^2) \\
& + \frac{m_c [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{1024\pi^6} \int dydz (y+z)(1-y-z) (s - \overline{m}_c^2)^2 \\
& - \frac{m_s [3\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{12288\pi^6} \int dydz (y+z)(1-y-z)^2 (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{384\pi^6} \int dydz yz(1-y-z) (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& + \frac{m_s \langle \bar{q}g_s \sigma Gq \rangle}{1536\pi^6} \int dydz yz(1-y-z) (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& + \frac{\langle \bar{q}q \rangle [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{144\pi^4} \int dydz yz(1-y-z) (s - \overline{m}_c^2) (5s - 3\overline{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}q \rangle [\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{144\pi^4} \int dydz (y+z) (s - \overline{m}_c^2) \\
& - \frac{5 [\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{576\pi^4} \int dydz yz (4s - 3\overline{m}_c^2) \\
& + \frac{\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{576\pi^4} \int dydz (y+z)(1-y-z) (4s - 3\overline{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle}{144\pi^4} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) \\
& - \frac{m_s m_c [36\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle - 2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle - 3\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{1728\pi^4} \int dy \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{2304\pi^4} \int dy \\
& - \frac{m_c \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{18\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{36\pi^2} \int dy y(1-y) \left[ 1 + \frac{s}{3} \delta(s - \tilde{m}_c^2) \right] \\
& - \frac{\langle \bar{q}g_s \sigma Gq \rangle [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{512\pi^4} \int dydz (y+z) \left[ 1 + \frac{s}{3} \delta(s - \overline{m}_c^2) \right] \\
& + \frac{\langle \bar{q}g_s \sigma Gq \rangle [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{256\pi^4} \int dy y(1-y) \left[ 1 + \frac{s}{3} \delta(s - \tilde{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle^2}{576\pi^4} \int dy \left( \frac{1-y}{y} + \frac{y}{1-y} \right) \delta(s - \tilde{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle [9\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \left( 1 + \frac{s}{2M^2} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle \langle \bar{s}g_s \sigma Gs \rangle}{6912\pi^4} \int dy \left( 1 + \frac{s}{2M^2} \right) \delta(s - \tilde{m}_c^2) , \tag{59}
\end{aligned}$$

$$\begin{aligned}
\hat{\rho}_{uus}^{11,0}(s) = & \frac{1}{122880\pi^8} \int dydz (y+z)(1-y-z)^4 (s-\bar{m}_c^2)^4 (7s-2\bar{m}_c^2) \\
& + \frac{m_s m_c}{18432\pi^8} \int dydz (1-y-z)^3 (s-\bar{m}_c^2)^4 \\
& - \frac{m_c [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{1152\pi^6} \int dydz (1-y-z)^2 (s-\bar{m}_c^2)^3 \\
& + \frac{m_s [3\langle \bar{s}s \rangle - 2\langle \bar{q}q \rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\bar{m}_c^2)^2 (5s-2\bar{m}_c^2) \\
& + \frac{m_c [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{512\pi^6} \int dydz (1-y-z) (s-\bar{m}_c^2)^2 \\
& - \frac{m_s [3\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{3072\pi^6} \int dydz (1-y-z)^2 (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{384\pi^6} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s \langle \bar{q}g_s \sigma Gq \rangle}{1536\pi^6} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{\langle \bar{q}q \rangle [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{144\pi^4} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}q \rangle [6\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{72\pi^4} \int dydz (s-\bar{m}_c^2) \\
& - \frac{5 [\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{1152\pi^4} \int dydz (y+z) (3s-2\bar{m}_c^2) \\
& + \frac{\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{288\pi^4} \int dydz (1-y-z) (3s-2\bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle}{144\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{1152\pi^4} \int dy \\
& - \frac{m_s m_c [36\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle - 2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle - 3\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{864\pi^4} \int dy \\
& - \frac{m_c \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{108\pi^2} \int dy \left[ 1 + \frac{s}{2} \delta(s-\tilde{m}_c^2) \right] \\
& - \frac{\langle \bar{q}g_s \sigma Gq \rangle [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{768\pi^4} \int dydz \left[ 1 + \frac{s}{2} \delta(s-\bar{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle^2}{576\pi^4} \int dy \left( \frac{1}{y} + \frac{1}{1-y} \right) \delta(s-\tilde{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle [9\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s-\tilde{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{q}g_s \sigma Gq \rangle \langle \bar{s}g_s \sigma Gs \rangle}{6912\pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s-\tilde{m}_c^2) , \tag{60}
\end{aligned}$$

$$\rho_{uuu}^{11,1}(s) = \rho_{sss}^{11,1}(s) \big|_{m_s \rightarrow 0, \langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle, \langle \bar{s}g_s \sigma Gs \rangle \rightarrow \langle \bar{q}g_s \sigma Gq \rangle} , \tag{61}$$

$$\hat{\rho}_{uuu}^{11,0}(s) = \hat{\rho}_{sss}^{11,0}(s) \big|_{m_s \rightarrow 0, \langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle, \langle \bar{s}g_s \sigma Gs \rangle \rightarrow \langle \bar{q}g_s \sigma Gq \rangle} , \tag{62}$$

$$\begin{aligned}
\rho_{ss}^{10,1}(s) = & \frac{1}{122880\pi^8} \int dydz yz(1-y-z)^4 (s-\overline{m}_c^2)^4 (8s-3\overline{m}_c^2) \\
& + \frac{1}{153600\pi^8} \int dydz yz(1-y-z)^5 (s-\overline{m}_c^2)^3 (18s^2-16s\overline{m}_c^2+3\overline{m}_c^4) \\
& + \frac{m_s m_c}{12288\pi^8} \int dydz (y+z)(1-y-z)^3 (s-\overline{m}_c^2)^4 \\
& + \frac{m_s m_c}{49152\pi^8} \int dydz (y+z)(1-y-z)^4 (s-\overline{m}_c^2)^3 (7s-3\overline{m}_c^2) \\
& - \frac{m_c \langle \bar{s}s \rangle}{768\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\overline{m}_c^2)^3 \\
& - \frac{m_c \langle \bar{s}s \rangle}{768\pi^6} \int dydz (y+z)(1-y-z)^3 (s-\overline{m}_c^2)^2 (2s-\overline{m}_c^2) \\
& - \frac{m_s \langle \bar{s}s \rangle}{256\pi^6} \int dydz yz(1-y-z)^2 (s-\overline{m}_c^2)^2 (2s-\overline{m}_c^2) \\
& + \frac{m_s \langle \bar{s}s \rangle}{128\pi^6} \int dydz yz(1-y-z)^3 (s-\overline{m}_c^2) (7s^2-8s\overline{m}_c^2+2\overline{m}_c^4) \\
& + \frac{11m_c \langle \bar{s}g_s \sigma Gs \rangle}{4096\pi^6} \int dydz (y+z)(1-y-z) (s-\overline{m}_c^2)^2 \\
& + \frac{11m_c \langle \bar{s}g_s \sigma Gs \rangle}{8192\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& - \frac{7m_c \langle \bar{s}g_s \sigma Gs \rangle}{8192\pi^6} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) (1-y-z)^2 (s-\overline{m}_c^2)^2 \\
& - \frac{m_c \langle \bar{s}g_s \sigma Gs \rangle}{4096\pi^6} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) (1-y-z)^3 (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& + \frac{11m_s \langle \bar{s}g_s \sigma Gs \rangle}{2048\pi^6} \int dydz yz(1-y-z) (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& - \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle}{256\pi^6} \int dydz yz(1-y-z)^2 (15s^2-20s\overline{m}_c^2+6\overline{m}_c^4) \\
& + \frac{3m_s \langle \bar{s}g_s \sigma Gs \rangle}{8192\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle^2}{48\pi^4} \int dydz yz(1-y-z) (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle^2}{24\pi^4} \int dydz (y+z) (s-\overline{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}s \rangle^2}{48\pi^4} \int dydz (y+z)(1-y-z) (4s-3\overline{m}_c^2)
\end{aligned}$$

$$\begin{aligned}
& -\frac{19\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{768\pi^4}\int dydz\,yz\,(4s-3\overline{m}_c^2) \\
& -\frac{\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{512\pi^4}\int dydz\,(y+z)(1-y-z)\,(4s-3\overline{m}_c^2) \\
& +\frac{89m_sm_c\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{3072\pi^4}\int dydz\,(y+z)\left[1+\frac{s}{3}\delta(s-\overline{m}_c^2)\right] \\
& -\frac{181m_sm_c\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{9216\pi^4}\int dy+\frac{3m_sm_c\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{512\pi^4}\int dydz\left(\frac{z}{y}+\frac{y}{z}\right) \\
& -\frac{3m_sm_c\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{256\pi^4}\int dydz\left(\frac{z}{y}+\frac{y}{z}\right)(1-y-z)\left[1+\frac{s}{3}\delta(s-\overline{m}_c^2)\right] \\
& -\frac{m_c\langle\bar{s}s\rangle^3}{36\pi^2}\int dy+\frac{m_s\langle\bar{s}s\rangle^3}{12\pi^2}\int dy\,y(1-y)\left[1+\frac{s}{3}\delta(s-\tilde{m}_c^2)\right] \\
& +\frac{43\langle\bar{s}g_s\sigma Gs\rangle^2}{18432\pi^4}\int dydz\,(y+z)\left[1+\frac{s}{3}\delta(s-\overline{m}_c^2)\right] \\
& +\frac{11\langle\bar{s}g_s\sigma Gs\rangle^2}{1024\pi^4}\int dy\,y(1-y)\left[1+\frac{s}{3}\delta(s-\tilde{m}_c^2)\right] \\
& +\frac{m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{768\pi^4}\int dydz\left(\frac{z}{y}+\frac{y}{z}\right)\left(1+\frac{s}{2M^2}\right)\delta(s-\overline{m}_c^2) \\
& -\frac{17m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{9216\pi^4}\int dy\left(\frac{1-y}{y}+\frac{y}{1-y}\right)\delta(s-\tilde{m}_c^2) \\
& +\frac{17m_sm_c\langle\bar{s}g_s\sigma Gs\rangle^2}{4608\pi^4}\int dy\left(1+\frac{s}{2M^2}\right)\delta(s-\tilde{m}_c^2), \tag{63}
\end{aligned}$$

$$\begin{aligned}
\tilde{\rho}_{sss}^{10,0}(s) &= \frac{1}{122880\pi^8}\int dydz\,(y+z)(1-y-z)^4\,(s-\overline{m}_c^2)^4\,(7s-2\overline{m}_c^2) \\
& -\frac{1}{614400\pi^8}\int dydz\,(y+z)(1-y-z)^5\,(s-\overline{m}_c^2)^3\,(28s^2-21s\overline{m}_c^2+3\overline{m}_c^4) \\
& +\frac{m_sm_c}{3072\pi^8}\int dydz\,(1-y-z)^3\,(s-\overline{m}_c^2)^4 \\
& -\frac{m_sm_c}{12288\pi^8}\int dydz\,(1-y-z)^4\,(s-\overline{m}_c^2)^3\,(3s-\overline{m}_c^2) \\
& -\frac{m_c\langle\bar{s}s\rangle}{192\pi^6}\int dydz\,(1-y-z)^2\,(s-\overline{m}_c^2)^3 \\
& +\frac{m_c\langle\bar{s}s\rangle}{1152\pi^6}\int dydz\,(1-y-z)^3\,(s-\overline{m}_c^2)^2\,(5s-2\overline{m}_c^2) \\
& -\frac{m_s\langle\bar{s}s\rangle}{768\pi^6}\int dydz\,(y+z)(1-y-z)^2\,(s-\overline{m}_c^2)^2\,(5s-2\overline{m}_c^2) \\
& -\frac{m_s\langle\bar{s}s\rangle}{256\pi^6}\int dydz\,(y+z)(1-y-z)^3\,(s-\overline{m}_c^2)\,(5s^2-5s\overline{m}_c^2+\overline{m}_c^4)
\end{aligned}$$



$$\begin{aligned}
& + \frac{11m_c \langle \bar{s}g_s \sigma G s \rangle}{1024\pi^6} \int dydz (1-y-z) (s - \bar{m}_c^2)^2 \\
& - \frac{11m_c \langle \bar{s}g_s \sigma G s \rangle}{2048\pi^6} \int dydz (1-y-z)^2 (s - \bar{m}_c^2) (2s - \bar{m}_c^2) \\
& - \frac{7m_c \langle \bar{s}g_s \sigma G s \rangle}{4096\pi^6} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) (1-y-z)^2 (s - \bar{m}_c^2)^2 \\
& + \frac{m_c \langle \bar{s}g_s \sigma G s \rangle}{2048\pi^6} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) (1-y-z)^3 (s - \bar{m}_c^2) (2s - \bar{m}_c^2) \\
& + \frac{m_s \langle \bar{s}g_s \sigma G s \rangle}{512\pi^6} \int dydz (y+z)(1-y-z)^2 (10s^2 - 12s\bar{m}_c^2 + 3\bar{m}_c^4) \\
& + \frac{11m_s \langle \bar{s}g_s \sigma G s \rangle}{1024\pi^6} \int dydz (y+z)(1-y-z) (s - \bar{m}_c^2) (2s - \bar{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle^2}{24\pi^4} \int dydz (y+z)(1-y-z) (s - \bar{m}_c^2) (2s - \bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle^2}{6\pi^4} \int dydz (s - \bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle^2}{24\pi^4} \int dydz (1-y-z) (3s - 2\bar{m}_c^2) \\
& - \frac{19 \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{768\pi^4} \int dydz (y+z) (3s - 2\bar{m}_c^2) \\
& - \frac{89m_s m_c \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{2304\pi^4} \int dydz \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] - \frac{199m_s m_c \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{2304\pi^4} \int dy \\
& + \frac{3m_s m_c \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{256\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma G s \rangle}{128\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) (1-y-z) \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& - \frac{m_c \langle \bar{s}s \rangle^3}{9\pi^2} \int dy + \frac{m_s \langle \bar{s}s \rangle^3}{18\pi^2} \int dy \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& + \frac{11 \langle \bar{s}g_s \sigma G s \rangle^2}{1536\pi^4} \int dy \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma G s \rangle^2}{1536\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) \left( 1 + \frac{s}{M^2} \right) \delta(s - \bar{m}_c^2) \\
& - \frac{17m_s m_c \langle \bar{s}g_s \sigma G s \rangle^2}{4608\pi^4} \int dy \left( \frac{1}{y} + \frac{1}{1-y} \right) \delta(s - \bar{m}_c^2) \\
& + \frac{125m_s m_c \langle \bar{s}g_s \sigma G s \rangle^2}{9216\pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s - \bar{m}_c^2) , \tag{64}
\end{aligned}$$

$$\begin{aligned}
\rho_{uss}^{10,1}(s) = & \frac{1}{122880\pi^8} \int dydz yz(1-y-z)^4 (s-\overline{m}_c^2)^4 (8s-3\overline{m}_c^2) \\
& + \frac{1}{153600\pi^8} \int dydz yz(1-y-z)^5 (s-\overline{m}_c^2)^3 (18s^2-16s\overline{m}_c^2+3\overline{m}_c^4) \\
& + \frac{m_s m_c}{18432\pi^8} \int dydz (y+z)(1-y-z)^3 (s-\overline{m}_c^2)^4 \\
& + \frac{m_s m_c}{73728\pi^8} \int dydz (y+z)(1-y-z)^4 (s-\overline{m}_c^2)^3 (7s-3\overline{m}_c^2) \\
& - \frac{m_c [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\overline{m}_c^2)^3 \\
& - \frac{m_c [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^3 (s-\overline{m}_c^2)^2 (2s-\overline{m}_c^2) \\
& - \frac{m_s [2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{384\pi^6} \int dydz yz(1-y-z)^2 (s-\overline{m}_c^2)^2 (2s-\overline{m}_c^2) \\
& + \frac{m_s \langle \bar{s}s \rangle}{192\pi^6} \int dydz yz(1-y-z)^3 (s-\overline{m}_c^2) (7s^2-8s\overline{m}_c^2+2\overline{m}_c^4) \\
& + \frac{11m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{12288\pi^6} \int dydz (y+z)(1-y-z) (s-\overline{m}_c^2)^2 \\
& + \frac{11m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{24576\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& - \frac{7m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{24576\pi^6} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) (1-y-z)^2 (s-\overline{m}_c^2)^2 \\
& - \frac{m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{12288\pi^6} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) (1-y-z)^3 (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& - \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle}{384\pi^6} \int dydz yz(1-y-z)^2 (15s^2-20s\overline{m}_c^2+6\overline{m}_c^4) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{8192\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& + \frac{m_s \langle \bar{q}g_s \sigma Gq \rangle}{384\pi^6} \int dydz yz(1-y-z) (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{2048\pi^6} \int dydz yz(1-y-z) (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{144\pi^4} \int dydz yz(1-y-z) (s-\overline{m}_c^2) (5s-3\overline{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}s \rangle [5\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{144\pi^4} \int dydz (y+z) (s-\overline{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{144\pi^4} \int dydz (y+z)(1-y-z) (4s-3\overline{m}_c^2)
\end{aligned}$$

$$\begin{aligned}
& - \frac{19 [\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{2304\pi^4} \int dydz yz (4s - 3\bar{m}_c^2) \\
& - \frac{\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle}{1536\pi^4} \int dydz (y+z)(1-y-z) (4s - 3\bar{m}_c^2) \\
& + \frac{m_s m_c [2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + 3\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + 5\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{576\pi^4} \int dydz (y+z) \left[1 + \frac{s}{3} \delta(s - \bar{m}_c^2)\right] \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{1024\pi^4} \int dydz (y+z) \left[1 + \frac{s}{3} \delta(s - \bar{m}_c^2)\right] \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{3072\pi^4} \int dy \\
& - \frac{m_s m_c [16\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + 15\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle - 5\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \\
& + \frac{m_s m_c [2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{576\pi^4} \int dydz \left(\frac{z}{y} + \frac{y}{z}\right) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{4608\pi^4} \int dydz \left(\frac{z}{y} + \frac{y}{z}\right) \\
& - \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{256\pi^4} \int dydz \left(\frac{z}{y} + \frac{y}{z}\right) (1-y-z) \left[1 + \frac{s}{3} \delta(s - \bar{m}_c^2)\right] \\
& - \frac{m_c \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{36\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{18\pi^2} \int dy y(1-y) \left[1 + \frac{s}{3} \delta(s - \bar{m}_c^2)\right] \\
& + \frac{13\langle \bar{s}g_s \sigma Gs \rangle [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{18432\pi^4} \int dydz (y+z) \left[1 + \frac{s}{3} \delta(s - \bar{m}_c^2)\right] \\
& + \frac{11\langle \bar{s}g_s \sigma Gs \rangle [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{3072\pi^4} \int dy y(1-y) \left[1 + \frac{s}{3} \delta(s - \bar{m}_c^2)\right] \\
& + \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{2304\pi^4} \int dydz \left(\frac{z}{y} + \frac{y}{z}\right) \left(1 + \frac{s}{2M^2}\right) \delta(s - \bar{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [5\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{3456\pi^4} \int dy \left(\frac{1-y}{y} + \frac{y}{1-y}\right) \delta(s - \bar{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{27648\pi^4} \int dy \left(\frac{1-y}{y} + \frac{y}{1-y}\right) \delta(s - \bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [7\langle \bar{q}g_s \sigma Gq \rangle - 2\langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \left(1 + \frac{s}{2M^2}\right) \delta(s - \bar{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{4608\pi^4} \int dy \left(1 + \frac{s}{2M^2}\right) \delta(s - \bar{m}_c^2) , \tag{65}
\end{aligned}$$

$$\begin{aligned}
\tilde{\rho}_{uss}^{10,0}(s) = & \frac{1}{122880\pi^8} \int dydz (y+z)(1-y-z)^4 (s-\overline{m}_c^2)^4 (7s-2\overline{m}_c^2) \\
& - \frac{1}{614400\pi^8} \int dydz (y+z)(1-y-z)^5 (s-\overline{m}_c^2)^3 (28s^2-21s\overline{m}_c^2+3\overline{m}_c^4) \\
& + \frac{m_s m_c}{4608\pi^8} \int dydz (1-y-z)^3 (s-\overline{m}_c^2)^4 \\
& - \frac{m_s m_c}{18432\pi^8} \int dydz (1-y-z)^4 (s-\overline{m}_c^2)^3 (3s-\overline{m}_c^2) \\
& - \frac{m_c [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{576\pi^6} \int dydz (1-y-z)^2 (s-\overline{m}_c^2)^3 \\
& + \frac{m_c [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{3456\pi^6} \int dydz (1-y-z)^3 (s-\overline{m}_c^2)^2 (5s-2\overline{m}_c^2) \\
& - \frac{m_s [2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{1152\pi^6} \int dydz (y+z)(1-y-z)^2 (s-\overline{m}_c^2)^2 (5s-2\overline{m}_c^2) \\
& - \frac{m_s \langle \bar{s}s \rangle}{384\pi^6} \int dydz (y+z)(1-y-z)^3 (s-\overline{m}_c^2) (5s^2-5s\overline{m}_c^2+\overline{m}_c^4) \\
& + \frac{11m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{3072\pi^6} \int dydz (1-y-z) (s-\overline{m}_c^2)^2 \\
& - \frac{11m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{6144\pi^6} \int dydz (1-y-z)^2 (s-\overline{m}_c^2) (2s-\overline{m}_c^2) \\
& - \frac{7m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{12288\pi^6} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) (1-y-z)^2 (s-\overline{m}_c^2)^2 \\
& + \frac{m_c [\langle \bar{q}g_s \sigma Gq \rangle + 2\langle \bar{s}g_s \sigma Gs \rangle]}{6144\pi^6} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) (1-y-z)^3 (s-\overline{m}_c^2) (2s-\overline{m}_c^2) \\
& + \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle}{768\pi^6} \int dydz (y+z)(1-y-z)^2 (10s^2-12s\overline{m}_c^2+3\overline{m}_c^4) \\
& + \frac{m_s \langle \bar{q}g_s \sigma Gq \rangle}{192\pi^6} \int dydz (y+z)(1-y-z) (s-\overline{m}_c^2) (2s-\overline{m}_c^2) \\
& + \frac{m_s [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{1024\pi^6} \int dydz (y+z)(1-y-z) (s-\overline{m}_c^2) (2s-\overline{m}_c^2) \\
& + \frac{\langle \bar{s}s \rangle [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{72\pi^4} \int dydz (y+z)(1-y-z) (s-\overline{m}_c^2) (2s-\overline{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [5\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{36\pi^4} \int dydz (s-\overline{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{72\pi^4} \int dydz (1-y-z) (3s-2\overline{m}_c^2)
\end{aligned}$$

$$\begin{aligned}
& - \frac{19 [\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{2304\pi^4} \int dydz (y+z) (3s - 2\bar{m}_c^2) \\
& - \frac{m_s m_c [2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + 3\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle + 5\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{432\pi^4} \int dydz \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{768\pi^4} \int dydz \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{768\pi^4} \int dy \\
& - \frac{m_s m_c [16\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + 15\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle - 5\langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{432\pi^4} \int dy \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{2304\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) \\
& + \frac{m_s m_c [2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}s \rangle \langle \bar{s}g_s \sigma Gs \rangle]}{288\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{384\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) (1-y-z) \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& - \frac{m_c \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{9\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{27\pi^2} \int dy \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& + \frac{11 \langle \bar{s}g_s \sigma Gs \rangle [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{4608\pi^4} \int dy \left[ 1 + \frac{s}{2} \delta(s - \bar{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{4608\pi^4} \int dydz \left( \frac{1}{y} + \frac{1}{z} \right) \left( 1 + \frac{s}{M^2} \right) \delta(s - \bar{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [5\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \left( \frac{1}{y} + \frac{1}{1-y} \right) \delta(s - \bar{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{13824\pi^4} \int dy \left( \frac{1}{y} + \frac{1}{1-y} \right) \delta(s - \bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [15\langle \bar{q}g_s \sigma Gq \rangle - 3\langle \bar{s}g_s \sigma Gs \rangle]}{1728\pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s - \bar{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{s}g_s \sigma Gs \rangle [\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{9216\pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s - \bar{m}_c^2), \tag{66}
\end{aligned}$$

$$\begin{aligned}
\rho_{uus}^{10,1}(s) &= \frac{1}{122880\pi^8} \int dydz yz(1-y-z)^4 (s - \bar{m}_c^2)^4 (8s - 3\bar{m}_c^2) \\
&+ \frac{1}{153600\pi^8} \int dydz yz(1-y-z)^5 (s - \bar{m}_c^2)^3 (18s^2 - 16s\bar{m}_c^2 + 3\bar{m}_c^4) \\
&+ \frac{m_s m_c}{36864\pi^8} \int dydz (y+z)(1-y-z)^3 (s - \bar{m}_c^2)^4 \\
&+ \frac{m_s m_c}{147456\pi^8} \int dydz (y+z)(1-y-z)^4 (s - \bar{m}_c^2)^3 (7s - 3\bar{m}_c^2) \\
&- \frac{m_c [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^2 (s - \bar{m}_c^2)^3 \\
&- \frac{m_c [2\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^3 (s - \bar{m}_c^2)^2 (2s - \bar{m}_c^2) \\
&- \frac{m_s [4\langle \bar{q}q \rangle - 3\langle \bar{s}s \rangle]}{768\pi^6} \int dydz yz(1-y-z)^2 (s - \bar{m}_c^2)^2 (2s - \bar{m}_c^2) \\
&+ \frac{m_s \langle \bar{s}s \rangle}{384\pi^6} \int dydz yz(1-y-z)^3 (s - \bar{m}_c^2) (7s^2 - 8s\bar{m}_c^2 + 2\bar{m}_c^4)
\end{aligned}$$

$$\begin{aligned}
& + \frac{11m_c [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{12288\pi^6} \int dydz (y+z)(1-y-z) (s - \bar{m}_c^2)^2 \\
& + \frac{11m_c [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{24576\pi^6} \int dydz (y+z) (1-y-z)^2 (s - \bar{m}_c^2) (5s - 3\bar{m}_c^2) \\
& - \frac{7m_c [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{24576\pi^6} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) (1-y-z)^2 (s - \bar{m}_c^2)^2 \\
& - \frac{m_c [2\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle]}{12288\pi^6} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) (1-y-z)^3 (s - \bar{m}_c^2) (5s - 3\bar{m}_c^2) \\
& - \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle}{768\pi^6} \int dydz yz(1-y-z)^2 (15s^2 - 20s\bar{m}_c^2 + 6\bar{m}_c^4) \\
& + \frac{m_s \langle \bar{q}g_s \sigma Gq \rangle}{8192\pi^6} \int dydz (y+z)(1-y-z)^2 (s - \bar{m}_c^2) (5s - 3\bar{m}_c^2) \\
& + \frac{m_s [2\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{768\pi^6} \int dydz yz(1-y-z) (s - \bar{m}_c^2) (5s - 3\bar{m}_c^2) \\
& + \frac{m_s \langle \bar{q}g_s \sigma Gq \rangle}{2048\pi^6} \int dydz yz(1-y-z) (s - \bar{m}_c^2) (5s - 3\bar{m}_c^2) \\
& + \frac{\langle \bar{q}q \rangle [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{144\pi^4} \int dydz yz(1-y-z) (s - \bar{m}_c^2) (5s - 3\bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}q \rangle [3\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{144\pi^4} \int dydz (y+z) (s - \bar{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{144\pi^4} \int dydz (y+z)(1-y-z) (4s - 3\bar{m}_c^2) \\
& - \frac{19 [\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{2304\pi^4} \int dydz yz (4s - 3\bar{m}_c^2) \\
& - \frac{\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{1536\pi^4} \int dydz (y+z)(1-y-z) (4s - 3\bar{m}_c^2) \\
& + \frac{m_s m_c [2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + 3\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{576\pi^4} \int dydz (y+z) \left[ 1 + \frac{s}{3} \delta (s - \bar{m}_c^2) \right] \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{1024\pi^4} \int dydz (y+z) \left[ 1 + \frac{s}{3} \delta (s - \bar{m}_c^2) \right] \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{3072\pi^4} \int dy \\
& - \frac{m_s m_c [18\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle - 2\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle - 3\langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle]}{1728\pi^4} \int dy \\
& + \frac{m_s m_c [2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle] \langle \bar{q}g_s \sigma Gq \rangle}{576\pi^4} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{4608\pi^4} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) \\
& - \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{256\pi^4} \int dydz \left( \frac{z}{y} + \frac{y}{z} \right) (1-y-z) \left[ 1 + \frac{s}{3} \delta (s - \bar{m}_c^2) \right] \\
& - \frac{m_c \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{36\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{36\pi^2} \int dy y(1-y) \left[ 1 + \frac{s}{3} \delta (s - \bar{m}_c^2) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{13\langle\bar{q}g_s\sigma Gq\rangle [\langle\bar{q}g_s\sigma Gq\rangle + 2\langle\bar{s}g_s\sigma Gs\rangle]}{18432\pi^4} \int dydz (y+z) \left[1 + \frac{s}{3}\delta(s - \overline{m}_c^2)\right] \\
& + \frac{11\langle\bar{q}g_s\sigma Gq\rangle [\langle\bar{q}g_s\sigma Gq\rangle + 2\langle\bar{s}g_s\sigma Gs\rangle]}{3072\pi^4} \int dy y(1-y) \left[1 + \frac{s}{3}\delta(s - \tilde{m}_c^2)\right] \\
& + \frac{m_s m_c \langle\bar{q}g_s\sigma Gq\rangle \langle\bar{s}g_s\sigma Gs\rangle}{2304\pi^4} \int dydz \left(\frac{z}{y} + \frac{y}{z}\right) \left(1 + \frac{s}{2M^2}\right) \delta(s - \overline{m}_c^2) \\
& - \frac{m_s m_c \langle\bar{q}g_s\sigma Gq\rangle [3\langle\bar{q}g_s\sigma Gq\rangle - \langle\bar{s}g_s\sigma Gs\rangle]}{3456\pi^4} \int dy \left(\frac{1-y}{y} + \frac{y}{1-y}\right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s m_c \langle\bar{q}g_s\sigma Gq\rangle \langle\bar{s}g_s\sigma Gs\rangle}{27648\pi^4} \int dy \left(\frac{1-y}{y} + \frac{y}{1-y}\right) \delta(s - \tilde{m}_c^2) \\
& + \frac{m_s m_c \langle\bar{q}g_s\sigma Gq\rangle [9\langle\bar{q}g_s\sigma Gq\rangle - 4\langle\bar{s}g_s\sigma Gs\rangle]}{3456\pi^4} \int dy \left(1 + \frac{s}{2M^2}\right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s m_c \langle\bar{q}g_s\sigma Gq\rangle \langle\bar{s}g_s\sigma Gs\rangle}{4608\pi^4} \int dy \left(1 + \frac{s}{2M^2}\right) \delta(s - \tilde{m}_c^2) , \tag{67}
\end{aligned}$$

$$\begin{aligned}
\tilde{\rho}_{uus}^{10,0}(s) &= \frac{1}{122880\pi^8} \int dydz (y+z)(1-y-z)^4 (s - \overline{m}_c^2)^4 (7s - 2\overline{m}_c^2) \\
& - \frac{1}{614400\pi^8} \int dydz (y+z)(1-y-z)^5 (s - \overline{m}_c^2)^3 (28s^2 - 21s\overline{m}_c^2 + 3\overline{m}_c^4) \\
& + \frac{m_s m_c}{9216\pi^8} \int dydz (1-y-z)^3 (s - \overline{m}_c^2)^4 \\
& - \frac{m_s m_c}{36864\pi^8} \int dydz (1-y-z)^4 (s - \overline{m}_c^2)^3 (3s - \overline{m}_c^2) \\
& - \frac{m_c [2\langle\bar{q}q\rangle + \langle\bar{s}s\rangle]}{576\pi^6} \int dydz (1-y-z)^2 (s - \overline{m}_c^2)^3 \\
& + \frac{m_c [2\langle\bar{q}q\rangle + \langle\bar{s}s\rangle]}{3456\pi^6} \int dydz (1-y-z)^3 (s - \overline{m}_c^2)^2 (5s - 2\overline{m}_c^2) \\
& - \frac{m_s [4\langle\bar{q}q\rangle - 3\langle\bar{s}s\rangle]}{2304\pi^6} \int dydz (y+z)(1-y-z)^2 (s - \overline{m}_c^2)^2 (5s - 2\overline{m}_c^2) \\
& - \frac{m_s \langle\bar{s}s\rangle}{768\pi^6} \int dydz (y+z)(1-y-z)^3 (s - \overline{m}_c^2) (5s^2 - 5s\overline{m}_c^2 + \overline{m}_c^4)
\end{aligned}$$

$$\begin{aligned}
& + \frac{11m_c [2\langle \bar{q}g_s\sigma Gq \rangle + \langle \bar{s}g_s\sigma Gs \rangle]}{3072\pi^6} \int dydz (1-y-z) (s-\bar{m}_c^2)^2 \\
& - \frac{11m_c [2\langle \bar{q}g_s\sigma Gq \rangle + \langle \bar{s}g_s\sigma Gs \rangle]}{6144\pi^6} \int dydz (1-y-z)^2 (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& - \frac{7m_c [2\langle \bar{q}g_s\sigma Gq \rangle + \langle \bar{s}g_s\sigma Gs \rangle]}{12288\pi^6} \int dydz \left(\frac{1}{y} + \frac{1}{z}\right) (1-y-z)^2 (s-\bar{m}_c^2)^2 \\
& + \frac{m_c [2\langle \bar{q}g_s\sigma Gq \rangle + \langle \bar{s}g_s\sigma Gs \rangle]}{6144\pi^6} \int dydz \left(\frac{1}{y} + \frac{1}{z}\right) (1-y-z)^3 (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s \langle \bar{s}g_s\sigma Gs \rangle}{1536\pi^6} \int dydz (y+z)(1-y-z)^2 (10s^2 - 12s\bar{m}_c^2 + 3\bar{m}_c^4) \\
& + \frac{m_s [2\langle \bar{q}g_s\sigma Gq \rangle - \langle \bar{s}g_s\sigma Gs \rangle]}{384\pi^6} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s \langle \bar{q}g_s\sigma Gq \rangle}{1024\pi^6} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{\langle \bar{q}q \rangle [\langle \bar{q}q \rangle + 2\langle \bar{s}s \rangle]}{72\pi^4} \int dydz (y+z)(1-y-z) (s-\bar{m}_c^2) (2s-\bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}q \rangle [3\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{36\pi^4} \int dydz (s-\bar{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q}q \rangle \langle \bar{s}s \rangle}{72\pi^4} \int dydz (1-y-z) (3s-2\bar{m}_c^2) \\
& - \frac{19 [\langle \bar{q}q \rangle \langle \bar{q}g_s\sigma Gq \rangle + \langle \bar{q}q \rangle \langle \bar{s}g_s\sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle]}{2304\pi^4} \int dydz (y+z) (3s-2\bar{m}_c^2) \\
& - \frac{m_s m_c [2\langle \bar{q}q \rangle \langle \bar{s}g_s\sigma Gs \rangle + 3\langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle]}{432\pi^4} \int dydz \left[1 + \frac{s}{2} \delta(s-\bar{m}_c^2)\right] \\
& - \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle}{768\pi^4} \int dydz \left[1 + \frac{s}{2} \delta(s-\bar{m}_c^2)\right] \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle}{768\pi^4} \int dy \\
& - \frac{m_s m_c [18\langle \bar{q}q \rangle \langle \bar{q}g_s\sigma Gq \rangle - 2\langle \bar{q}q \rangle \langle \bar{s}g_s\sigma Gs \rangle - 3\langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle]}{432\pi^4} \int dy \\
& + \frac{m_s m_c [2\langle \bar{q}q \rangle - \langle \bar{s}s \rangle] \langle \bar{q}g_s\sigma Gq \rangle}{288\pi^4} \int dydz \left(\frac{1}{y} + \frac{1}{z}\right) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle}{2304\pi^4} \int dydz \left(\frac{1}{y} + \frac{1}{z}\right) \\
& + \frac{m_s m_c \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle}{384\pi^4} \int dydz \left(\frac{1}{y} + \frac{1}{z}\right) (1-y-z) \left[1 + \frac{s}{2} \delta(s-\bar{m}_c^2)\right] \\
& - \frac{m_c \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9\pi^2} \int dy + \frac{m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{54\pi^2} \int dy \left[1 + \frac{s}{2} \delta(s-\bar{m}_c^2)\right]
\end{aligned}$$



$$\begin{aligned}
& + \frac{11 \langle \bar{q} g_s \sigma G q \rangle [\langle \bar{q} g_s \sigma G q \rangle + 2 \langle \bar{s} g_s \sigma G s \rangle]}{4608 \pi^4} \int dy \left[ 1 + \frac{s}{2} \delta(s - \tilde{m}_c^2) \right] \\
& - \frac{m_s m_c \langle \bar{q} g_s \sigma G q \rangle \langle \bar{s} g_s \sigma G s \rangle}{4608 \pi^4} \int dy dz \left( \frac{1}{y} + \frac{1}{z} \right) \left( 1 + \frac{s}{M^2} \right) \delta(s - \overline{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{q} g_s \sigma G q \rangle [3 \langle \bar{q} g_s \sigma G q \rangle - \langle \bar{s} g_s \sigma G s \rangle]}{1728 \pi^4} \int dy \left( \frac{1}{y} + \frac{1}{1-y} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{q} g_s \sigma G q \rangle \langle \bar{s} g_s \sigma G s \rangle}{13824 \pi^4} \int dy \left( \frac{1}{y} + \frac{1}{1-y} \right) \delta(s - \tilde{m}_c^2) \\
& + \frac{m_s m_c \langle \bar{q} g_s \sigma G q \rangle [9 \langle \bar{q} g_s \sigma G q \rangle - \langle \bar{s} g_s \sigma G s \rangle]}{1728 \pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s m_c \langle \bar{q} g_s \sigma G q \rangle \langle \bar{s} g_s \sigma G s \rangle}{9216 \pi^4} \int dy \left( 1 + \frac{s}{M^2} \right) \delta(s - \tilde{m}_c^2), \tag{68}
\end{aligned}$$

$$\rho_{uuu}^{10,1}(s) = \rho_{sss}^{10,1}(s) |_{m_s \rightarrow 0, \langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle, \langle \bar{s} g_s \sigma G s \rangle \rightarrow \langle \bar{q} g_s \sigma G q \rangle}, \tag{69}$$

$$\tilde{\rho}_{uuu}^{10,0}(s) = \tilde{\rho}_{sss}^{10,0}(s) |_{m_s \rightarrow 0, \langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle, \langle \bar{s} g_s \sigma G s \rangle \rightarrow \langle \bar{q} g_s \sigma G q \rangle}, \tag{70}$$

where  $\int dy dz = \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz$ ,  $\int dy = \int_{y_i}^{y_f} dy$ ,  $y_f = \frac{1+\sqrt{1-4m_c^2/s}}{2}$ ,  $y_i = \frac{1-\sqrt{1-4m_c^2/s}}{2}$ ,  $z_i = \frac{ym_c^2}{ys-m_c^2}$ ,  $\overline{m}_c^2 = \frac{(y+z)m_c^2}{yz}$ ,  $\tilde{m}_c^2 = \frac{m_c^2}{y(1-y)}$ ,  $\int_{y_i}^{y_f} dy \rightarrow \int_0^1 dy$ ,  $\int_{z_i}^{1-y} dz \rightarrow \int_0^{1-y} dz$  when the  $\delta$  functions  $\delta(s - \overline{m}_c^2)$  and  $\delta(s - \tilde{m}_c^2)$  appear.

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